回路設計のためのコンパクトモデルの基本と先端MOSFETモデルへの拡張

東京工業大学
2007年11月19日

広島大学先端物質科学研究科
広島大学HiSIM研究センター
三浦道子

目次

はじめに

p/n Junction
Shockleyの電流方程式

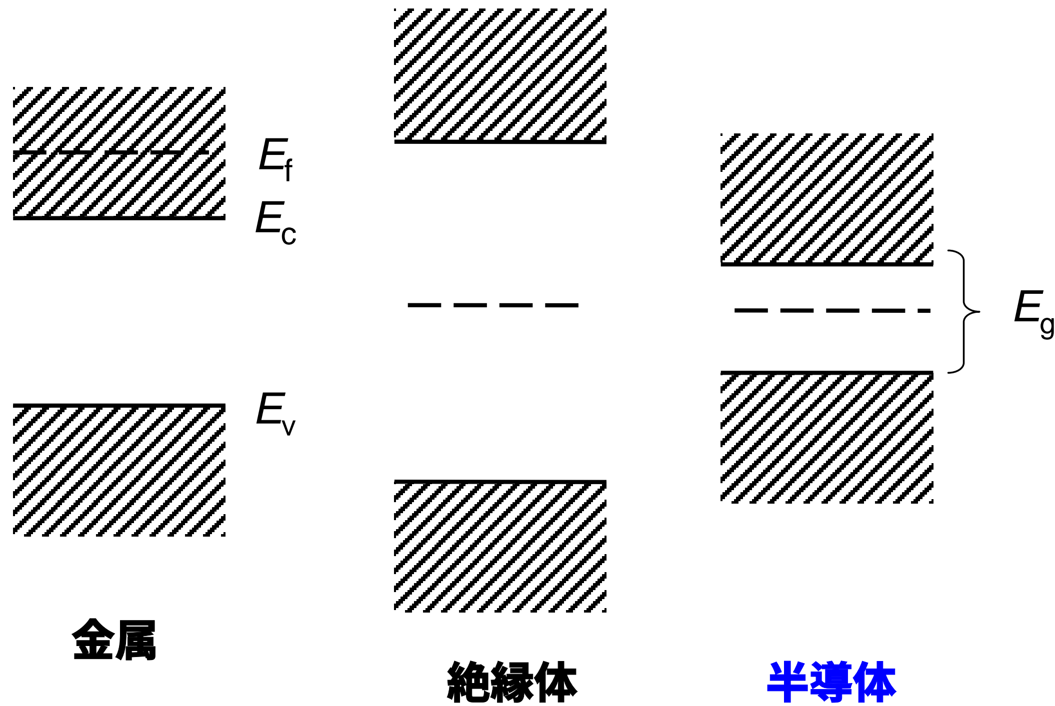
基礎編：

回路モデルとは
デバイスの基本原理
解析式の導出

応用編：

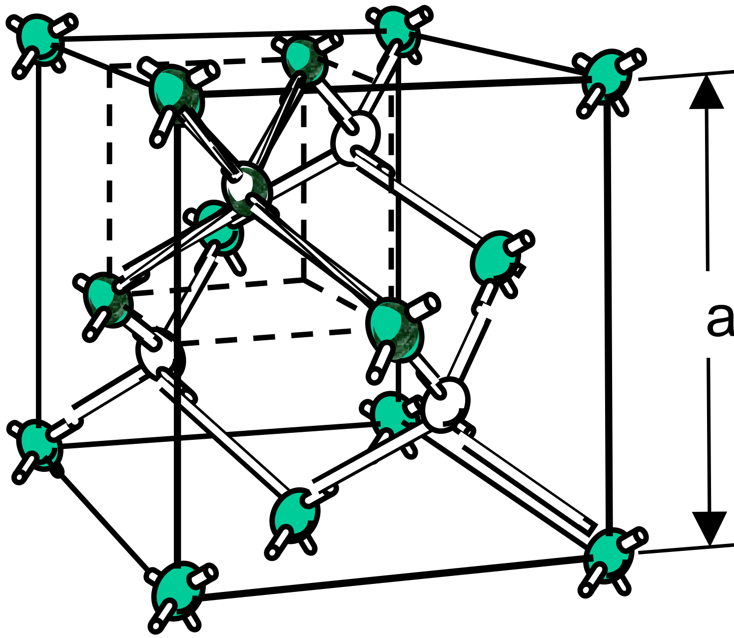
先端デバイスの特性
- 短チャネル効果
- Non-Quasi-Static効果
- ノイズ
- 高調波ひずみ
- Y-パラメタ特性

Wilsonのモデル

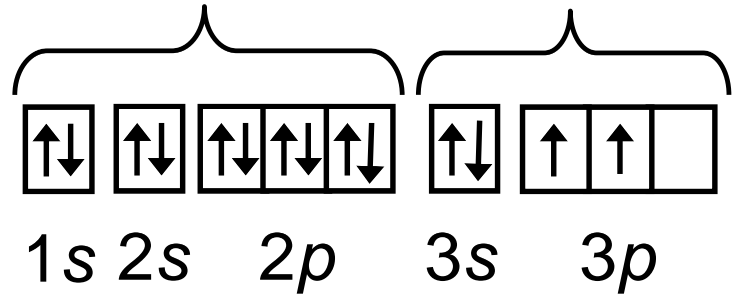


Bandgap: E_g
Fermi Level: E_f

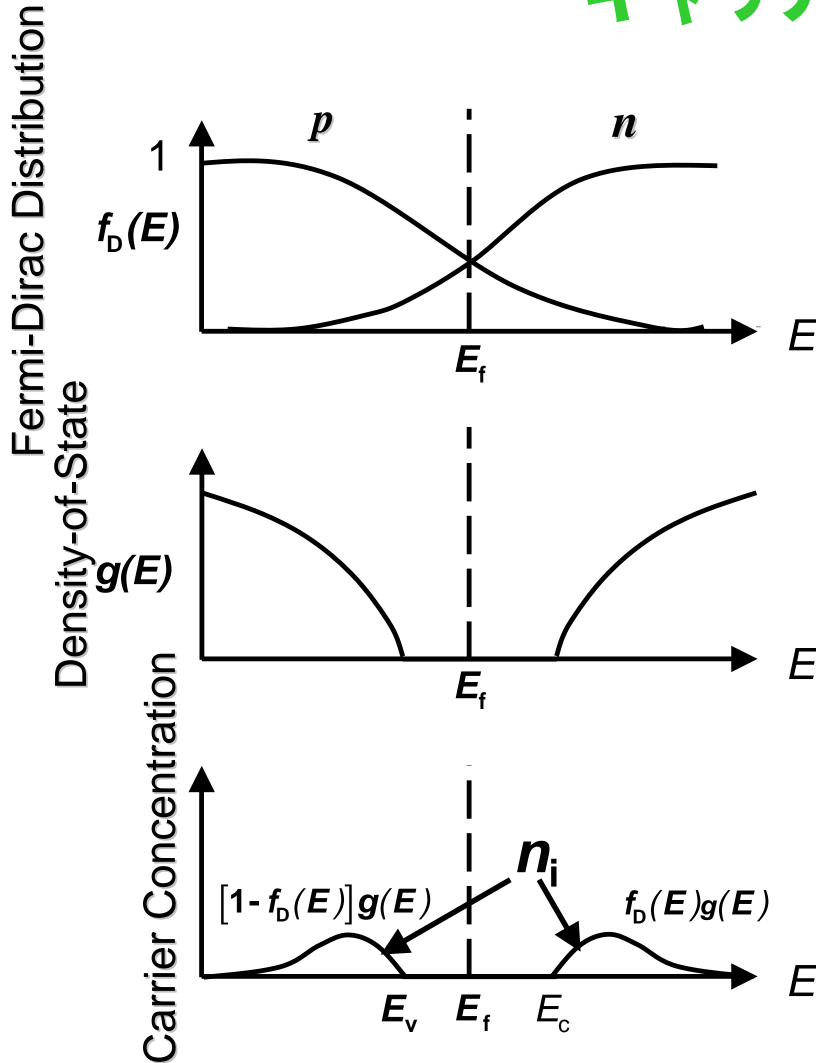
半導体結晶



共有結合



キャリア濃度



$$n = \int_{E_c}^{E_{top}} g(E) f_D(E)\, dE$$

$$p = \int_{E_{bottom}}^{E_v} g(E) (1 - f_D(E))\, dE$$

$$n = N_c \exp\left(- \frac{E_c - E_{fn}}{kT} \right)$$

$$p = N_v \exp\left(- \frac{E_{fp} - E_v}{kT} \right)$$

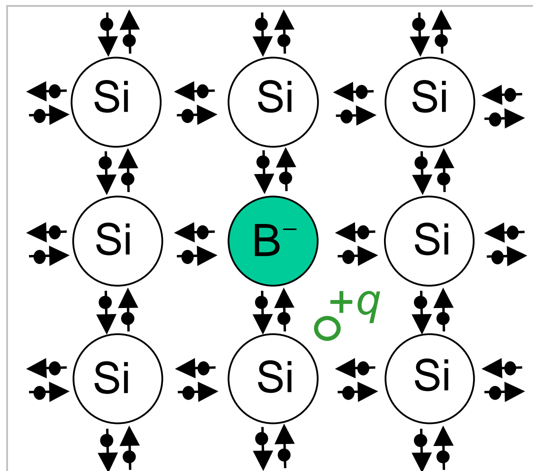
N_c : effective density of state

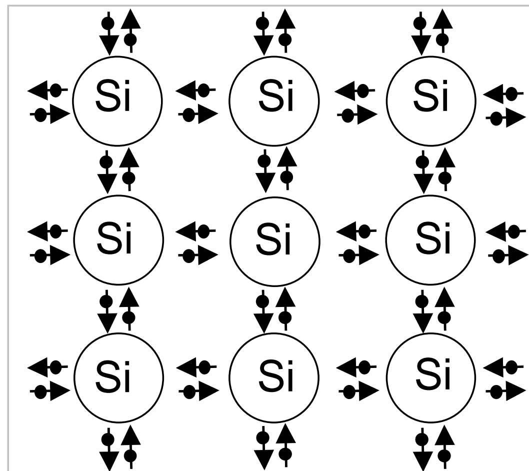
N_v: “

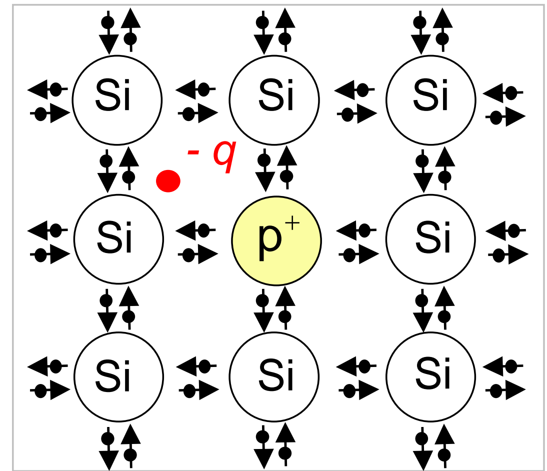
n_i (真性キャリア濃度) =1.4x10^{10}/cm^3

デバイス材料としての半導体

p型シリコン　　**真性シリコン**　　**n型シリコン**

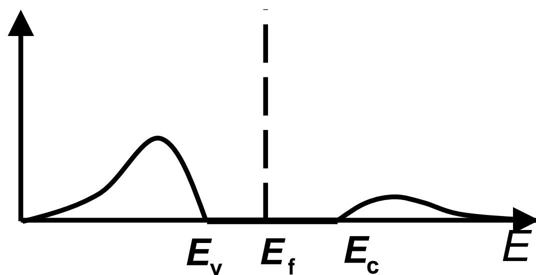


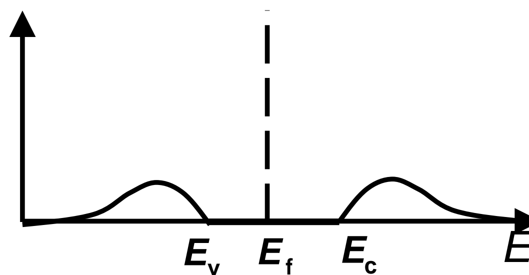


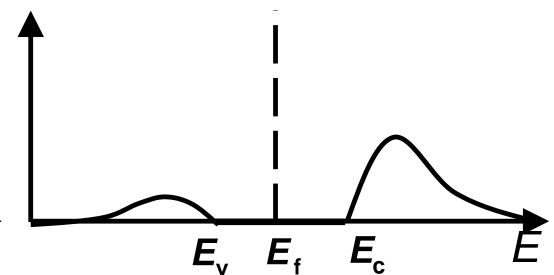


+q: ホール　　**-q: 電子**

n_i (真性キャリア濃度) =1.4x10^{10}/cm^3



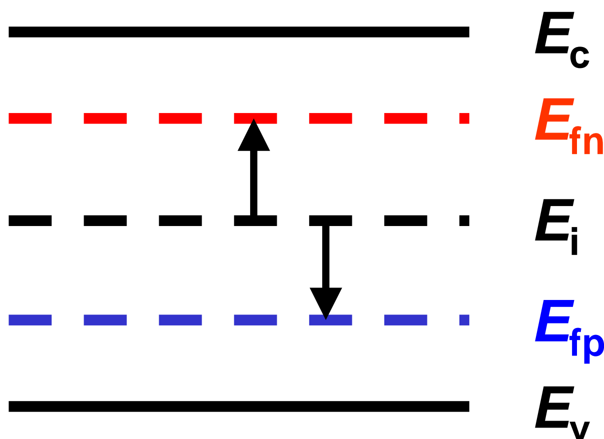




Fermi順位が変化

$$n = N_c \exp\left(-\frac{E_c - E_{fn}}{kT}\right) = n_i \exp(-q(\phi_{fn} - \phi_i)/kT)$$

$$p = N_v \exp\left(-\frac{E_{fp} - E_v}{kT}\right) = n_i \exp(-q(\phi_i - \phi_{fp})/kT)$$



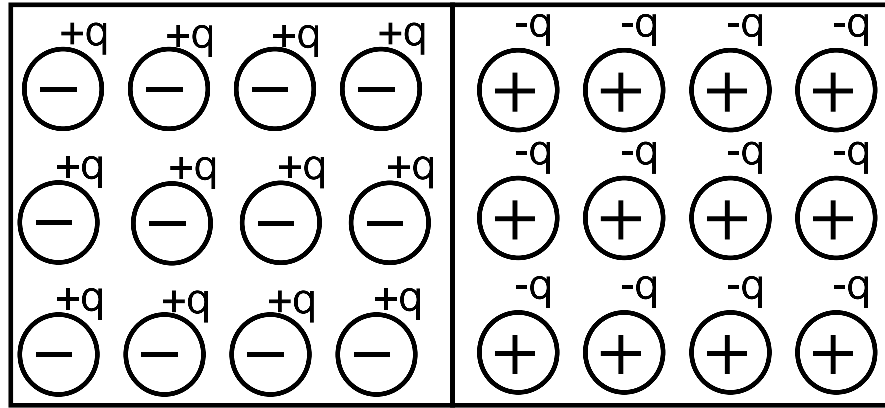
$E = -q\phi$

$N_c N_v \exp(E_g/kT) = n_i^2$

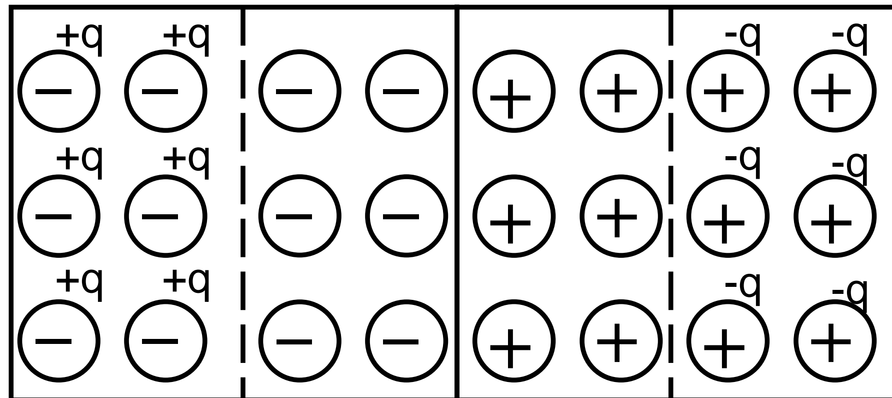
平衡状態：$pn = n_i^2$

p/n接合

p/n Junction



Equilibrium Condition



空乏層 (depletion region)

Fermi準位

p-type *n*-type

E_0

qX $q\Phi_{fp}$ $q\Phi_{fn}$

E_c E_{fn} E_{fp} E_v

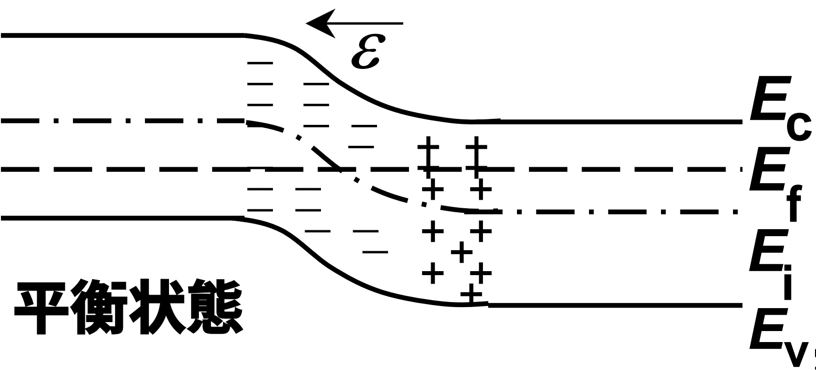
p/n接合

E_c E_{fn} E_{fp} E_v

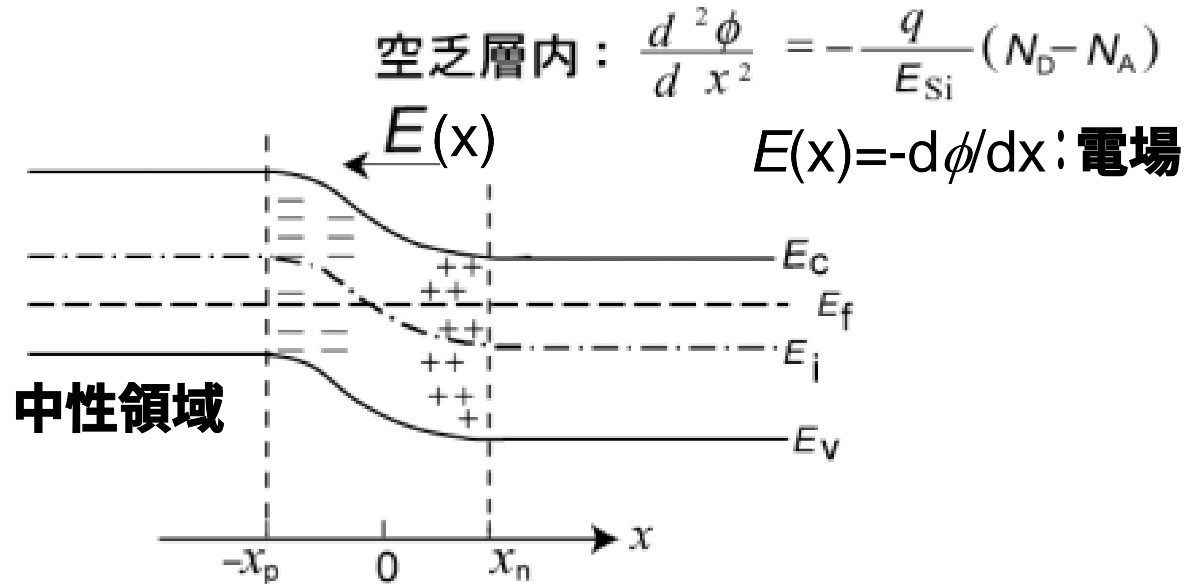
空乏層 (Depletion Region) の形成

-電場の強さ

-空乏層幅



Poisson方程式



(i) $0 \leq x \leq x_n$

$$E(x) = -\frac{qN_D}{\varepsilon_{si}}(x_n - x) \quad ; \quad \phi(x) = \phi_n - \frac{qN_D}{2\varepsilon_{si}}(x_n - x)^2$$

(ii) $-x_p \leq x \leq 0$

$$E(x) = -\frac{qN_A}{\varepsilon_{si}}(x + x_p) \quad ; \quad \phi(x) = \phi_p + \frac{qN_A}{2\varepsilon_{si}}(x + x_p)^2$$

空乏層幅：$x_n + x_p = \left[\frac{2\varepsilon_{si}}{q}\phi_i\left(\frac{1}{N_D} + \frac{1}{N_A}\right)\right]^{\frac{1}{2}}$; $\phi_i = \phi_n - \phi_p$

(Built-in Potential)

デバイスの基本方程式

-Poisson:

$$\nabla^2 \phi = -\frac{q}{\varepsilon_S}(N_D - N_A + p - n)$$

$$n = n_i \exp \frac{q(\phi - \phi_n)}{kT}$$

$$p = n_i \exp \frac{q(\phi_p - \phi)}{kT}$$

-Current Density:

$$j_n = -q\,\mu_n n \frac{\phi}{y} + qD_n \nabla n$$

$$j_p = -q\,\mu_p p \frac{\phi}{y} - qD_p \nabla p$$

-Continuity:

$$\frac{\partial n}{\partial t} = G_n - R_n + \frac{1}{q}\nabla j_n = 0$$

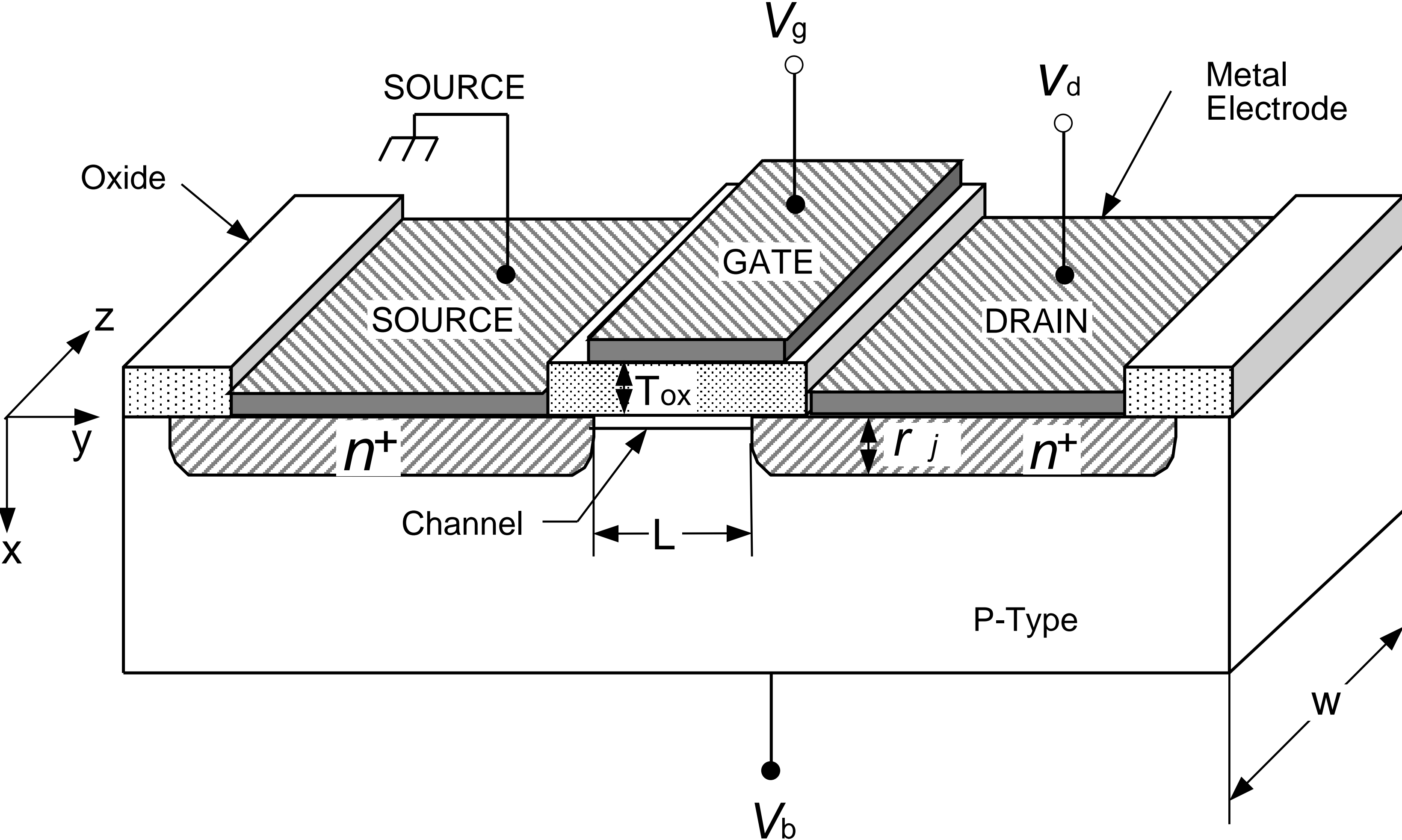
$$\frac{\partial p}{\partial t} = G_p - R_p - \frac{1}{q}\nabla j_p = 0$$

回路モデルとは

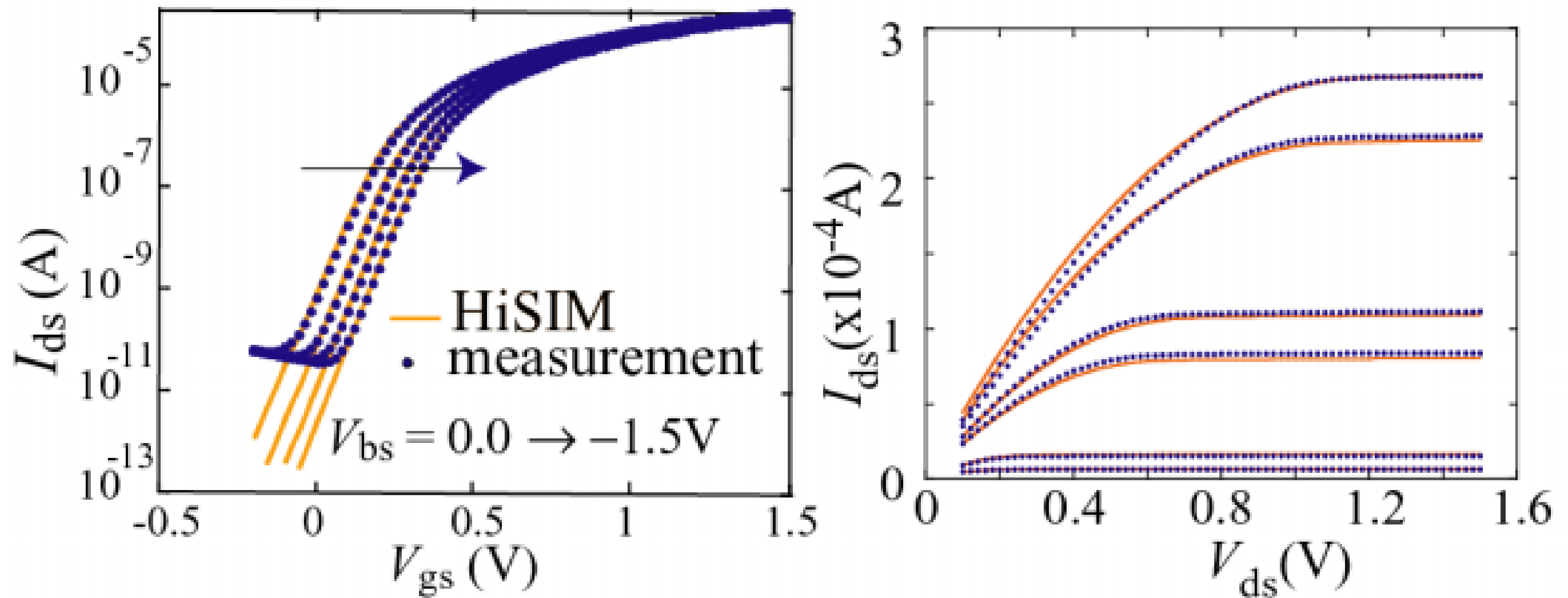
デバイス特性を解析式で記述したもの

- ✓ デバイス特性がコンパクトに記述されている
- ✓ すべての観測される現象がモデル化されている
- ✓ デバイス特性と回路性能の関係が予測できる
- ✓ 大規模回路がシミュレーションできる

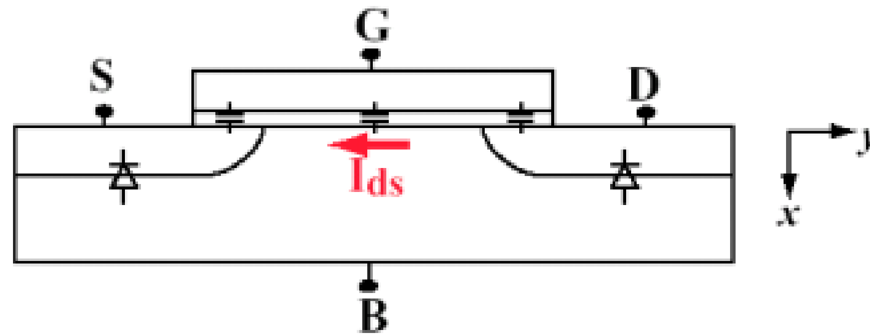
MOSFET

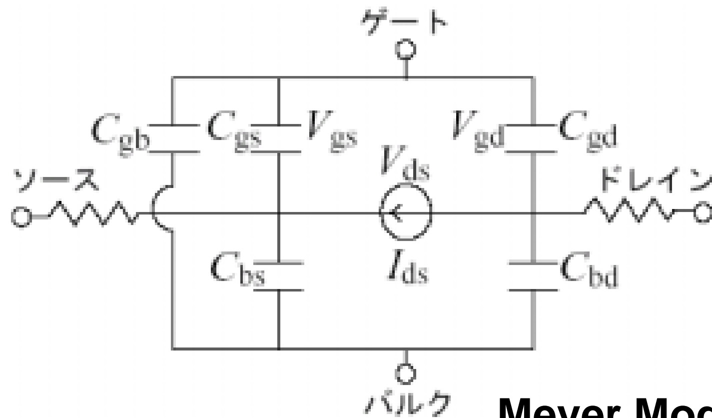


電流の測定値



MOSFET用回路モデル





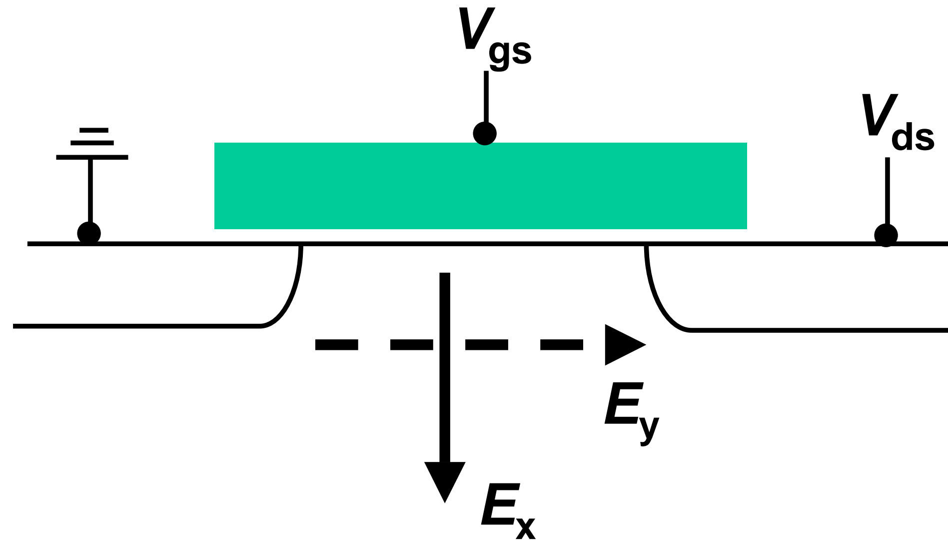
Meyer Model

J. E. Meyer, RCA Rev., vol. 32, p. 42, 1971.

$$I_{ds} = \mu \frac{W}{L} C_{ox} \left[(V_{gs} - V_{th}) V_{ds} - \frac{1}{2} V_{ds}^2 \right]$$ ⇨ **ドリフト近似**

C. T. Sah, IEEE TED, ED11, 324, 1964.

MOSFETの特徴

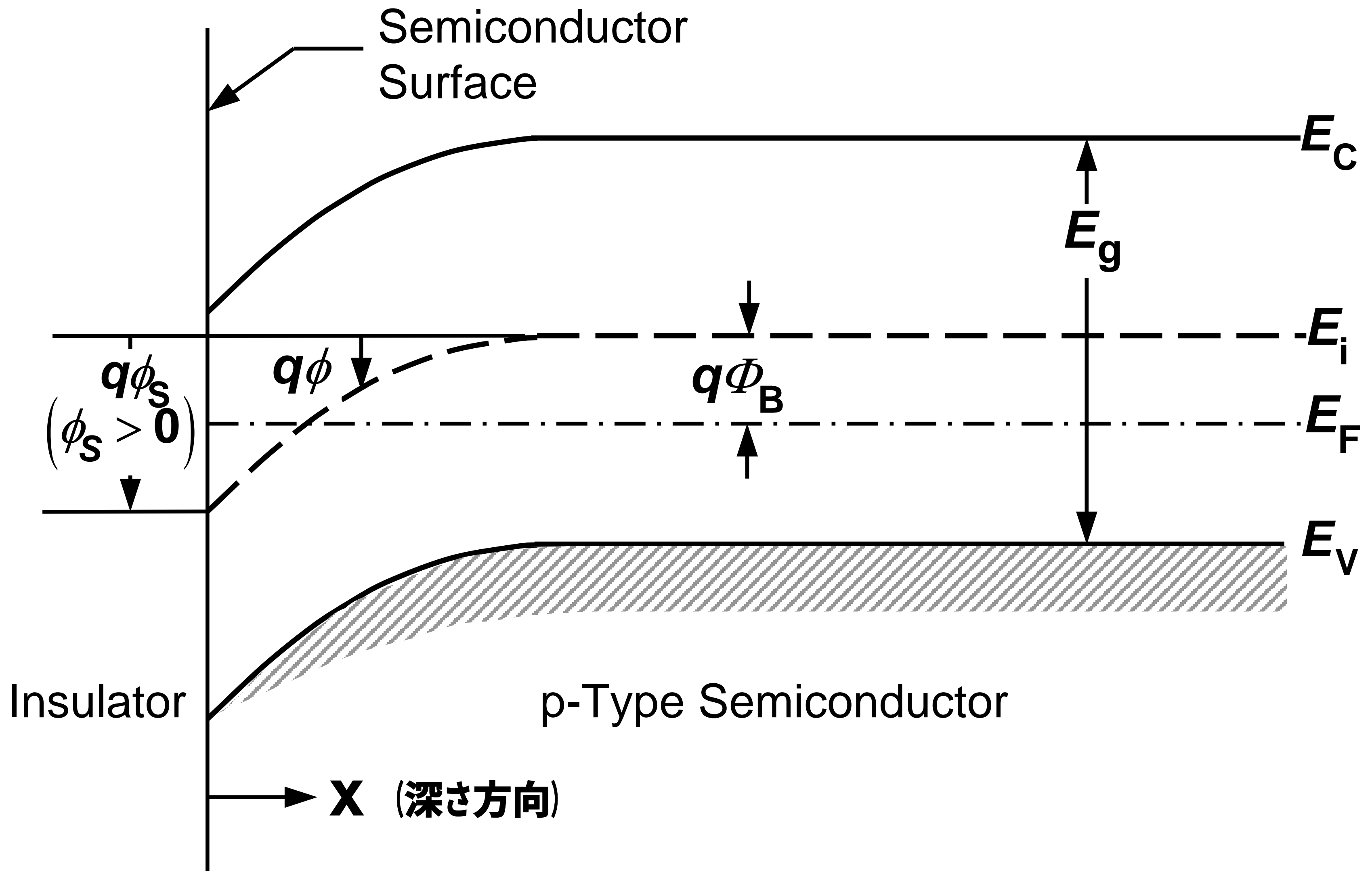


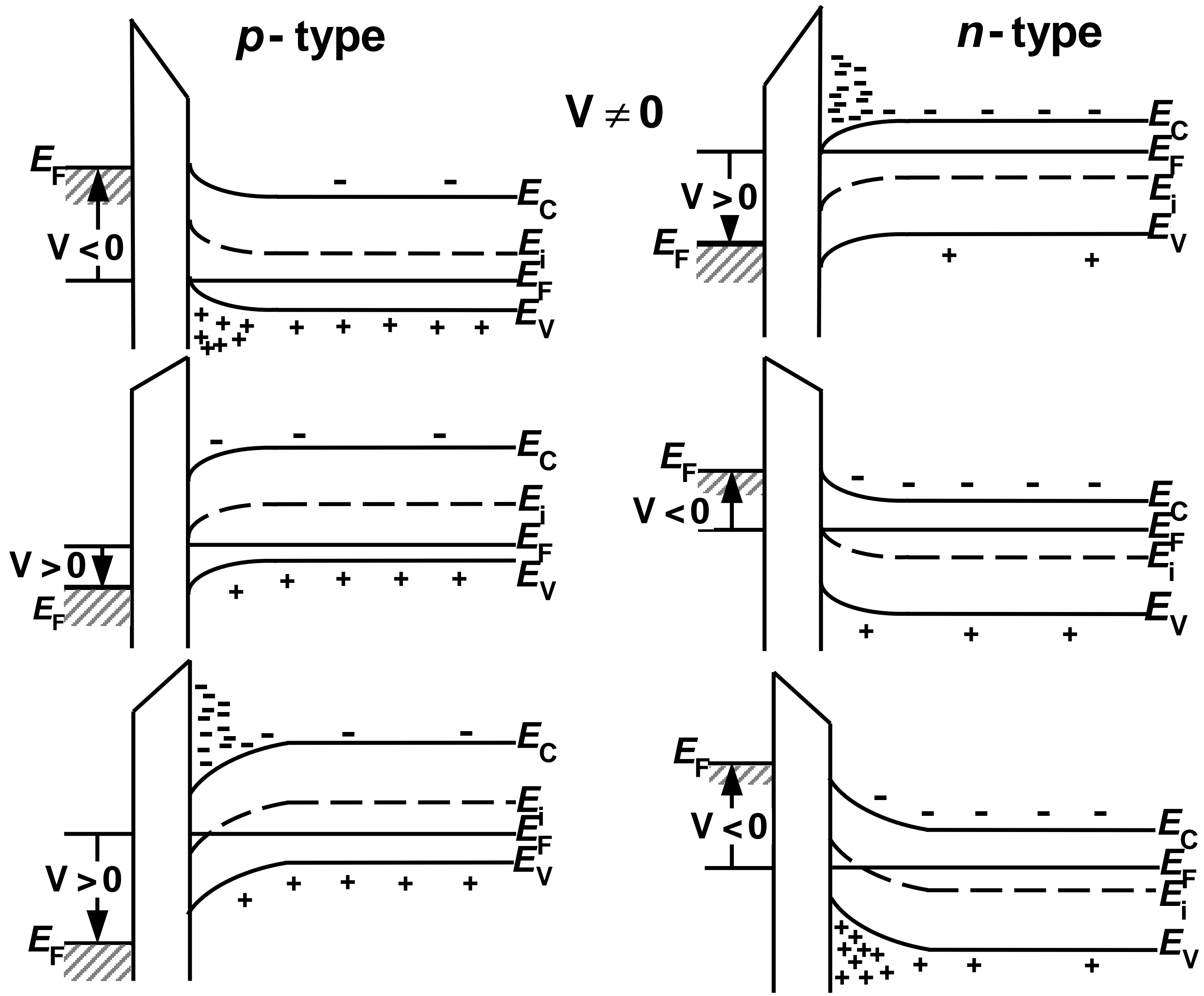
電流： $I \propto qnv$

- q:電荷
- n:キャリア濃度
- v:キャリア速度

V_{gs}：キャリアの濃度を決める
V_{ds}：キャリアの速さを決める

Vgsに対するポテンシャル分布




p- type
n- type
V ≠ 0
E_F
V < 0
E_C
E_i
E_F
E_V
V > 0
E_F
E_C
E_i
E_F
E_V
V > 0
E_F
E_C
E_i
E_F
E_V
V > 0
E_F
E_C
E_F
E_i
E_V
E_F
V < 0
E_C
E_F
E_i
E_V
E_F
V < 0
E_C
E_F
E_i
E_V

デバイスの基本方程式

-Poisson: $$\nabla^2\phi = -\frac{q}{\varepsilon_S}(N_D - N_A + p - n)$$

$$n = n_i \exp\frac{q(\phi - \phi_n)}{kT}$$

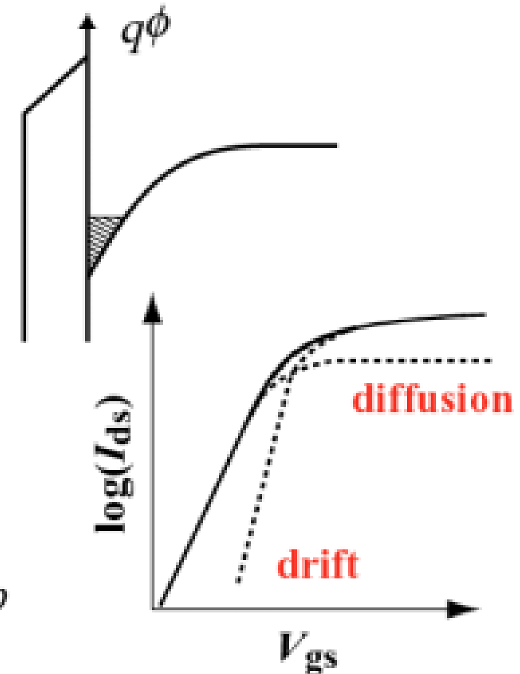
$$p = n_i \exp\frac{q(\phi_p - \phi)}{kT}$$

-Current Density: $$j_n = -q\,\mu_n n \frac{\phi}{y} + qD_n \nabla n$$

$$j_p = -q\,\mu_p p \frac{\phi}{y} - qD_p \nabla p$$

-Continuity: $$\frac{\partial n}{\partial t} = G_n - R_n + \frac{1}{q}\nabla j_n = 0$$

$$\frac{\partial p}{\partial t} = G_p - R_p - \frac{1}{q}\nabla j_p = 0$$



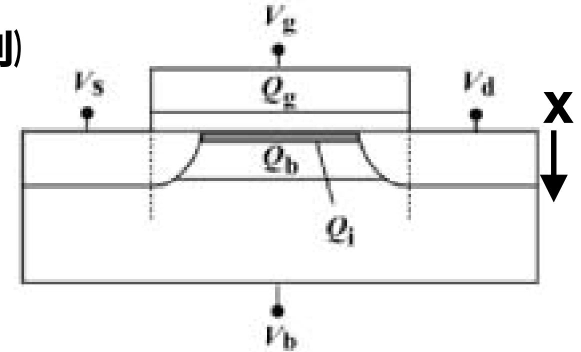
Poisson方程式:ϕ_s

$$\frac{d^2\phi}{dx^2} = -\frac{\rho(x)}{\varepsilon_s}$$

(Gradual Channel近似:深さ方向のみ考慮する)

(平衡状態:電荷中性の法則)

$$\rho(x) = q(N_D - N_A + p - n)$$
$$N_D - N_A = n_{p0} - p_{p0}$$
$$p - n = p_{p0}\exp(-\beta\phi) - n_{p0}\exp(\beta\phi)$$



$$\frac{\partial^2\phi}{\partial x^2} = -\frac{q}{\varepsilon_S}\int\left\{p_{p0}\left[\exp(-\beta\phi)-1\right]-n_{p0}\left[\exp(\beta\phi)-1\right]\right\}$$

$$\int_0^{\partial\phi/\partial x}\left(\frac{\partial\phi}{\partial x}\right)d\left(\frac{\partial\phi}{\partial x}\right) = -\frac{q}{\varepsilon_S}\int_0^{\phi}\left\{p_{p0}\left[\exp(-\beta\phi)-1\right]-n_{p0}\left[\exp(\beta\phi)-1\right]\right\}d\phi$$

$$E_S^2 = \left(\frac{2kT}{q}\right)^2\left(\frac{qp_{p0}\beta}{2\varepsilon_{si}}\right)\left\{\left[\exp(-\beta\phi)+\beta\phi+1\right]+\frac{n_{p0}}{p_{p0}}\left[\exp(\beta\phi)-\beta\phi-1\right]\right\}$$

Gauss's Law: $E_{ox}\varepsilon_{ox} = E_s\varepsilon_s = -Q_s$ 　Q_S: 基板中の全電荷

$$E_{ox} = (V_{gs} - V_{fb} - \phi_S)/T_{ox}$$

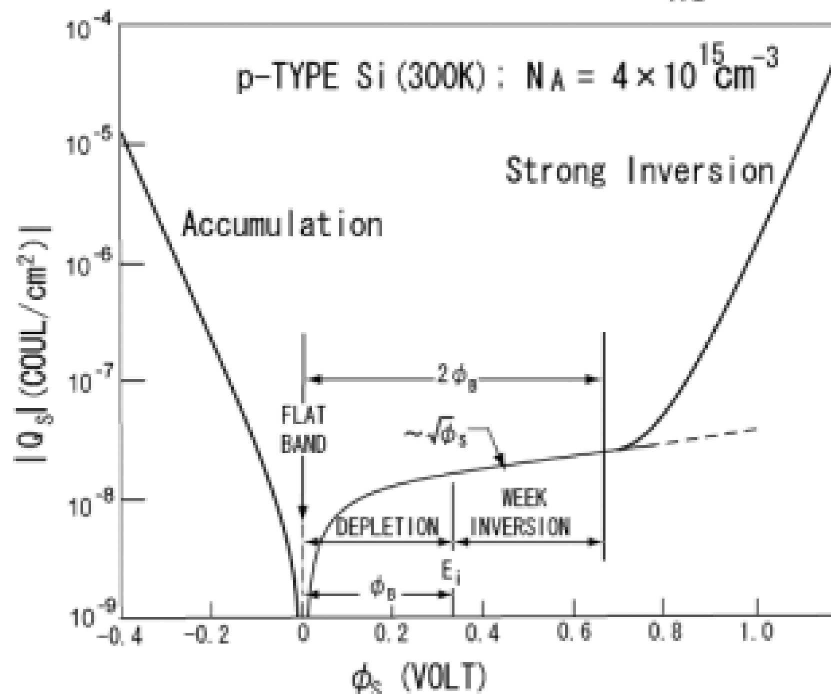
MOSキャパシタンス

$$-Q_S = C_{ox}\left(V'_G - \phi_S(y)\right) = \sqrt{\frac{2\varepsilon_s q N_{sub}}{\beta}} \Big[\exp\{-\beta(\phi_S(y) - V_{bs})\} + \beta(\phi_S(y) - V_{bs}) - 1$$

$$+ \frac{n_{p0}}{p_{p0}}\left\{\exp\left(\beta(\phi_S(y) - \phi_f(y))\right) - \exp\left(\beta(V_{bs} - \phi_f(y))\right)\right\}\Big]^{-\frac{1}{2}}$$

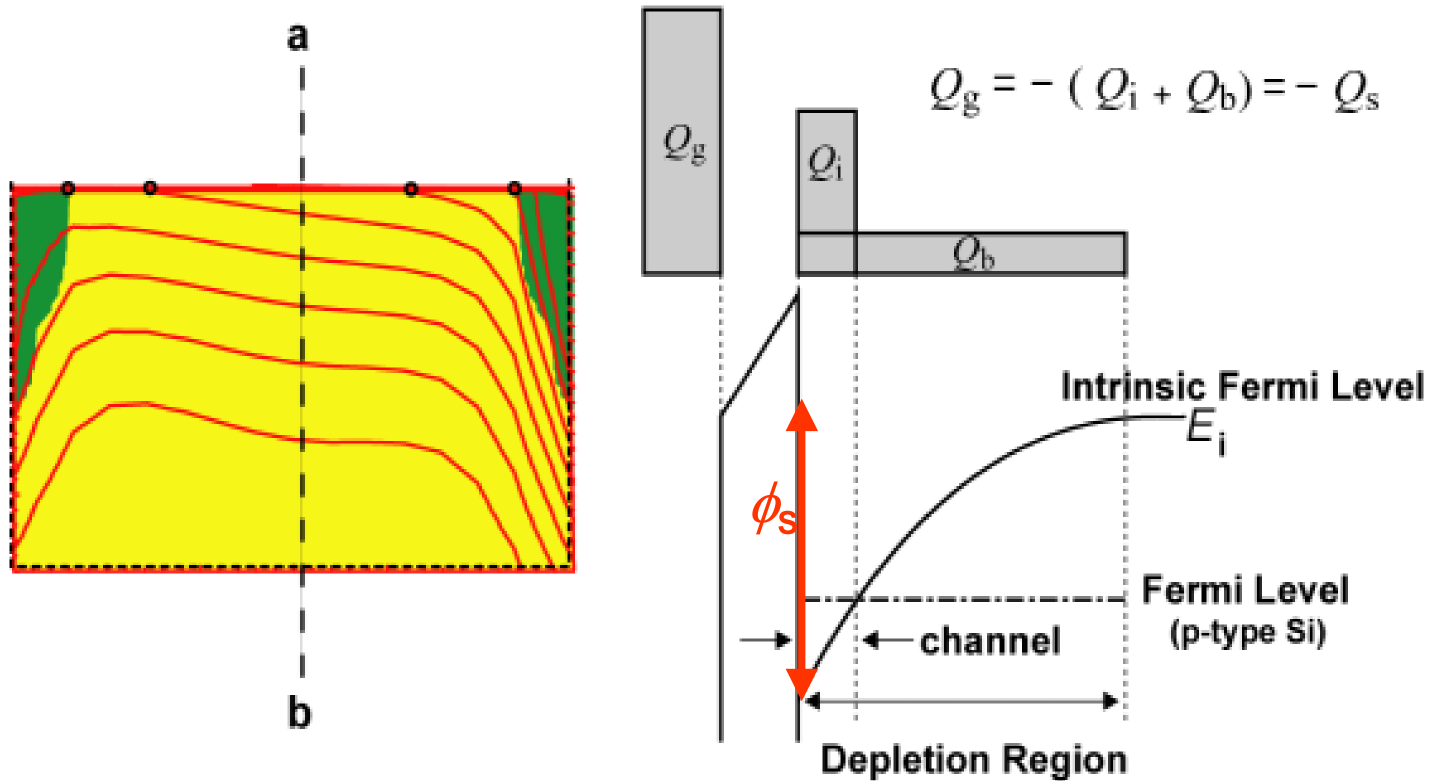
$$V'_G = V_{gs} - V_{fb}$$

$$\beta = \frac{q}{kT}$$



C. G. B. Garrett and W. H. Brattain, Phys. Rev., vol. 99, p. 376, 1955.

基板に誘起される全電荷



バルク電荷密度：Q_b=sqrt($\beta\phi_s$)

反転層電荷密度：Q_i=function(ϕ_s)←charge-sheet近似

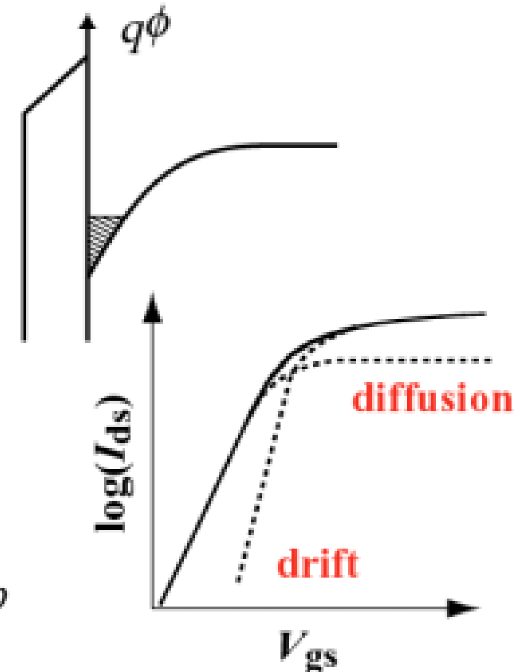
デバイスの基本方程式

-Poisson:
$$\nabla^2\phi = -\frac{q}{\varepsilon_S}(N_D - N_A + p - n)$$

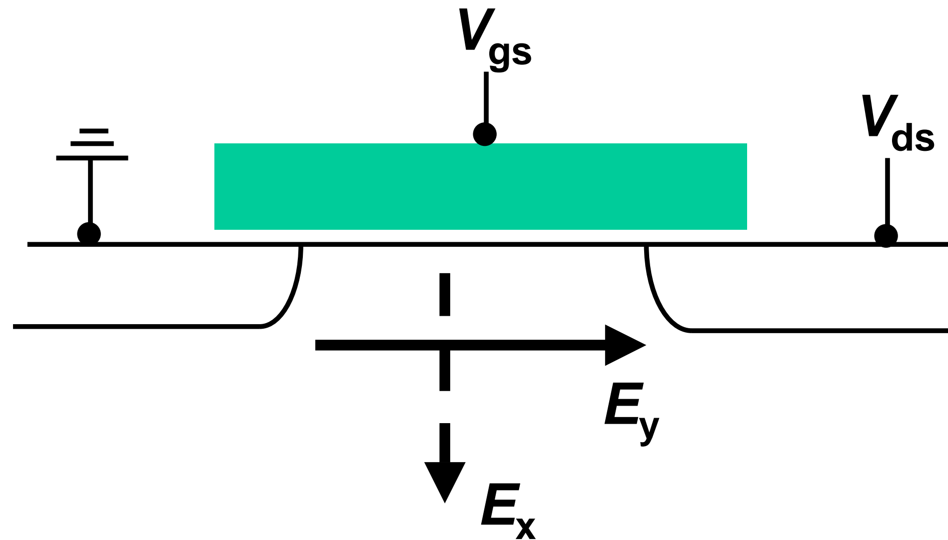
$$n = n_i \exp\frac{q(\phi - \phi_n)}{kT}$$
$$p = n_i \exp\frac{q(\phi_p - \phi)}{kT}$$

-Current Density:
$$j_n = -q\,\mu_n n \frac{\phi}{y} + qD_n \nabla n$$
$$j_p = -q\,\mu_p p \frac{\phi}{y} - qD_p \nabla p$$

-Continuity:
$$\frac{\partial n}{\partial t} = G_n - R_n + \frac{1}{q}\nabla j_n = 0$$
$$\frac{\partial p}{\partial t} = G_p - R_p - \frac{1}{q}\nabla j_p = 0$$



ドリフト近似



電流：$I \propto qnv$

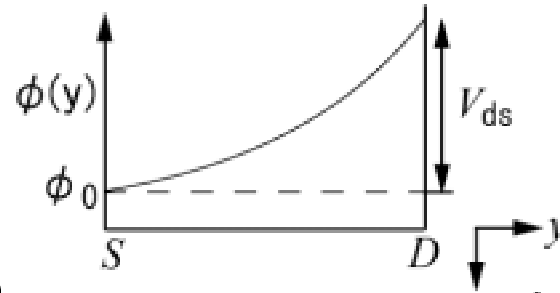
- q:電荷
- n:キャリア濃度
- v:キャリア速度

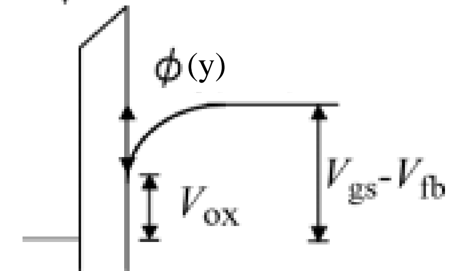
V_{gs}：キャリアの濃度を決める
V_{ds}：キャリアの速さを決める

$$I_{\mathrm{ds}} = W\mu q n \frac{d\phi}{dy}$$

$$I_{\mathrm{ds}} dy = W\mu q n d\phi$$

$$\phi(y) = \phi_0 + V(y)$$





Gaussの法則：$E_{\mathrm{ox}}\varepsilon_{\mathrm{ox}} = E_{\mathrm{Si}}\varepsilon_{\mathrm{Si}} = -Q_{\mathrm{S}}$

$$\frac{V_{\mathrm{gs}} - V_{\mathrm{fb}} - \phi(\mathrm{y})}{T_{\mathrm{ox}}}\varepsilon_{\mathrm{ox}} = -Q_{\mathrm{n}} - Q_{\mathrm{b}}$$

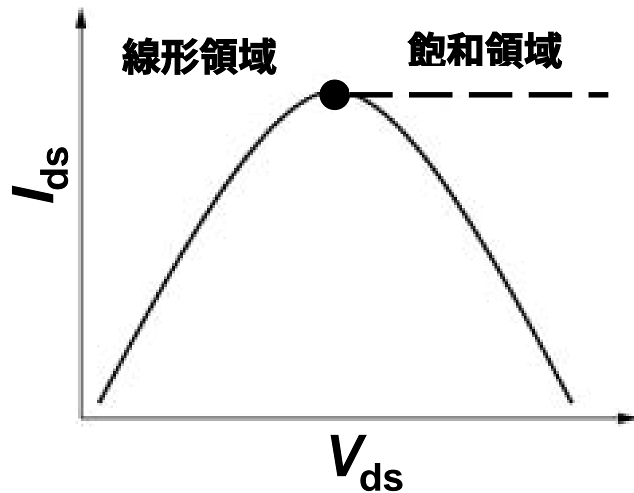
$$Q_{\mathrm{n}} = qn = -\left[V_{\mathrm{gs}} - V_{\mathrm{fb}} - \phi_0 - V(y)\right]C_{\mathrm{ox}} + \sqrt{2\varepsilon_{\mathrm{Si}} q N_{\mathrm{sub}}\left[\phi_0 + V(y)\right]}$$

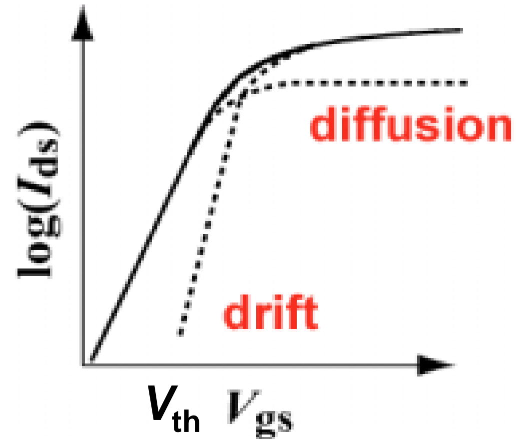
$$\int_0^L I_{\mathrm{ds}} dy = W\mu \int_0^{V_{\mathrm{ds}}} Q_{\mathrm{n}} dV \Rightarrow I_{\mathrm{ds}} = \mu \frac{W}{L} C_{\mathrm{ox}}\left[\left(V_{gs} - V_{\mathrm{th}}\right)V_{\mathrm{ds}} - \frac{1}{2}V_{\mathrm{ds}}^2\right]$$

$$\phi_0 = 2\Phi_{\mathrm{B}} ;\ V_{\mathrm{th}} = 2\Phi_{\mathrm{B}} + \frac{\sqrt{2E_{\mathrm{Si}} q N_{\mathrm{sub}} 2\Phi_{\mathrm{B}}}}{C_{\mathrm{ox}}}$$

ドリフト近似の問題点

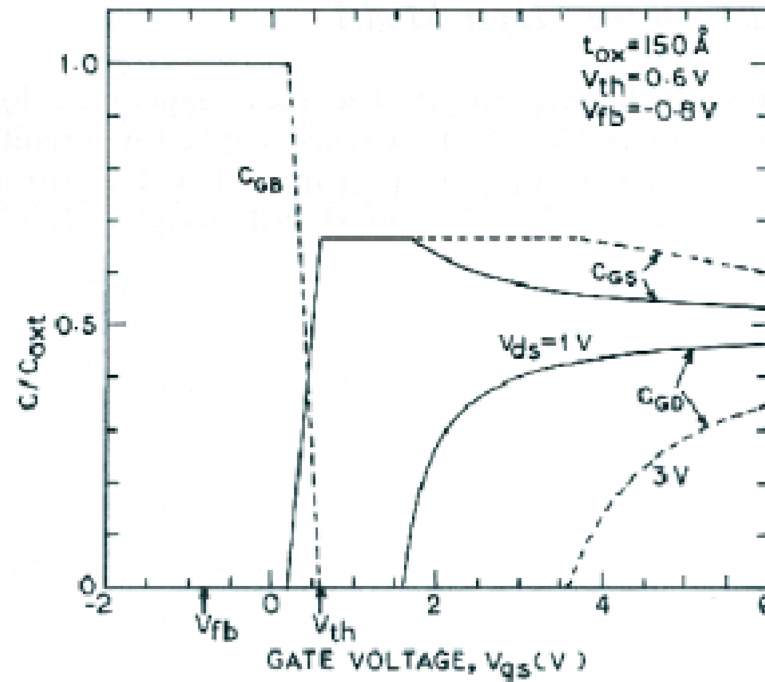
$$I_{ds} = \mu \frac{W}{L} C_{ox} \left[\left(V_{gs} - V_{th} \right) V_{ds} - \frac{1}{2} {V_{ds}}^2 \right]$$





キャパシタンスにおける問題

$$C_{gs} = \frac{2}{3} WLC_{ox}\left[1 - \frac{(V_{gs} - V_{th})^2}{(V_{gd} + V_{gs} - 2V_{th})^2} \right]$$



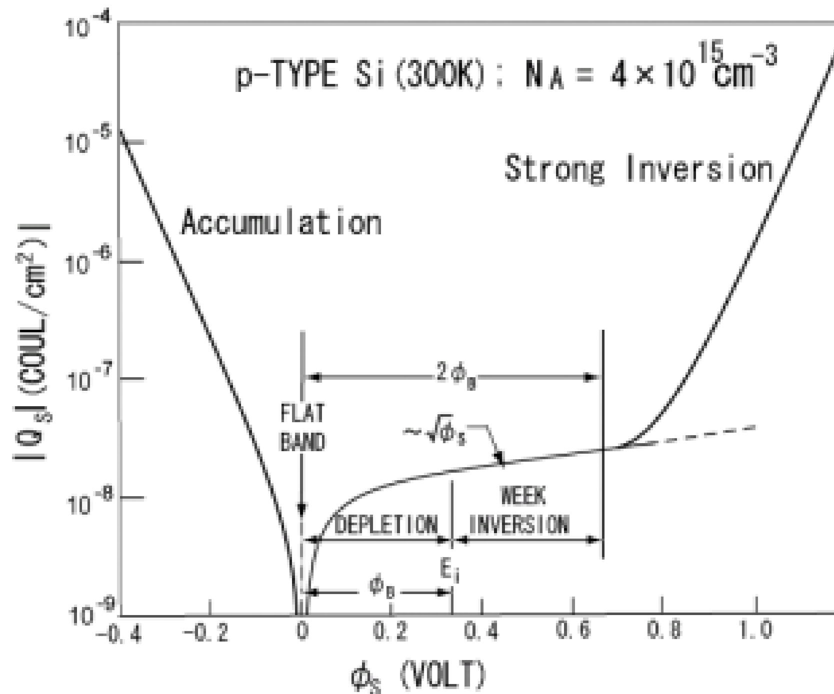
MOSキャパシタンス

$$-Q_S = C_{ox}\left(V'_G - \phi_S(y)\right) = \sqrt{\frac{2\varepsilon_s q N_{sub}}{\beta}}\left[\exp\{-\beta(\phi_S(y) - V_{bs})\} + \beta(\phi_S(y) - V_{bs}) - 1 + \frac{n_{p0}}{p_{p0}}\left\{\exp\left(\beta(\phi_S(y) - \phi_f(y))\right) - \exp\left(\beta(V_{bs} - \phi_f(y))\right)\right\}\right]^{-\frac{1}{2}}$$

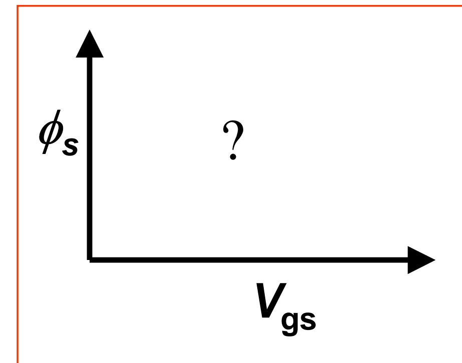
$$V'_G = V_{gs} - V_{fb}$$

$$\beta = \frac{q}{kT}$$

ϕ_f: 擬フェルミ(0→V_{ds})

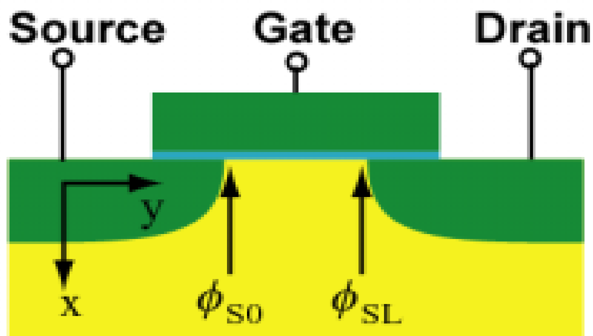


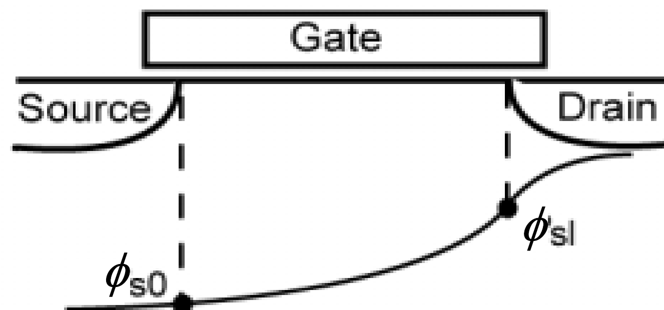
C. G. B. Garrett and W. H. Brattain, Phys. Rev., vol. 99, p. 376, 1955.

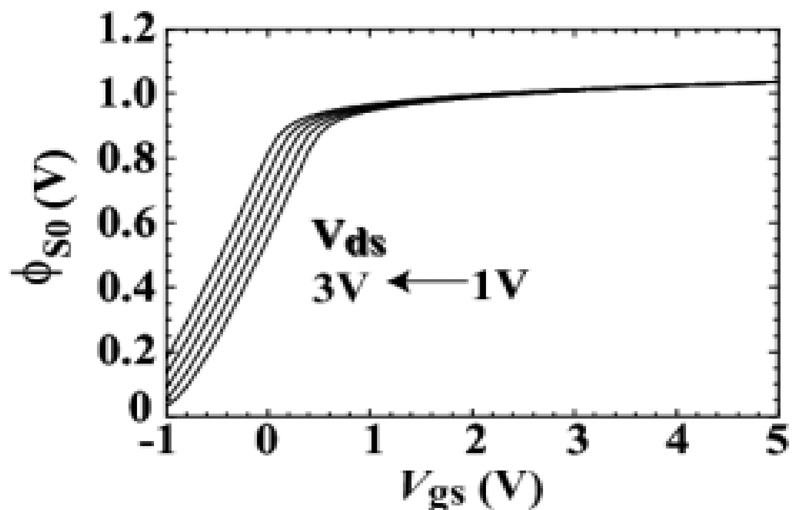


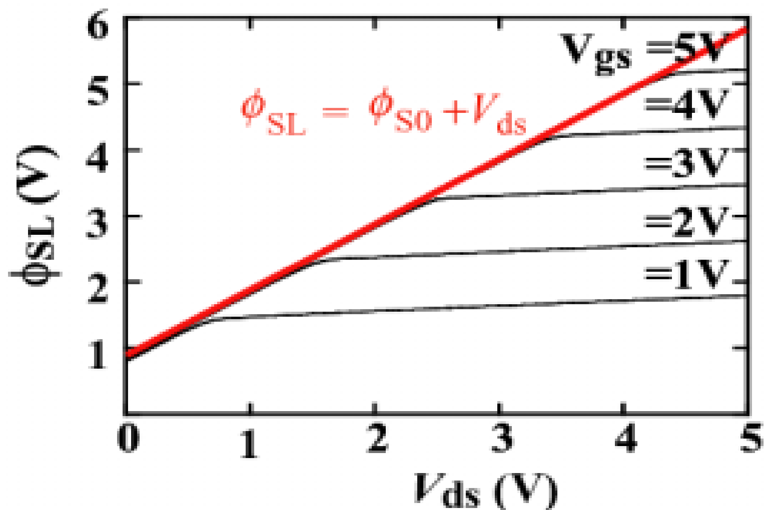
反復計算によって解く

表面ポテンシャルの計算結果









ϕ_{SL}: Gradual-Channel近似の終わり

ドリフト・拡散モデルの電流式導出

$$I_{ds}(y) = W_{eff}\, q\, \mu\, n(y)\left(-\frac{d\phi_S(y)}{dy} + \frac{1}{\beta}\frac{d\ln n(y)}{dy}\right)$$

$$I_{ds} = \int_0^L I_{ds}(y)\,dy$$

$$\left(\begin{array}{c} \dfrac{d\ln n(y)}{dy} = \dfrac{1}{n(y)}\dfrac{dn(y)}{dy} \\[2em] \dfrac{I_{ds}}{W_{eff}\, q\mu} + n(y)\beta\dfrac{d\phi_s(y)}{dy} = \dfrac{dn(y)}{dy} \\[1em] -C_{ox}\left[V_G - \phi_s(y)\right] = Q_B + qn(y) \end{array}\right)$$

電流はポテンシャルの関数

ドレイン電流式

$$I_{ds} = W_{eff}\, q\, \mu\, n(y)\left(-\frac{d\phi_S(y)}{dy} + \frac{1}{\beta}\frac{d\ln n(y)}{dy}\right)$$

$$I_{ds} = \frac{W_{eff}}{L_{eff}}\mu\frac{IDD}{\beta}$$

$$IDD = C_{ox}(\beta V_G' + 1)(\phi_{SL} - \phi_{S0}) - \frac{\beta}{2}C_{ox}(\phi_{SL}{}^2 - \phi_{S0}{}^2)$$
$$- \frac{2}{3}\sqrt{\frac{2\varepsilon_s q N_{sub}}{\beta}}\,[\{\beta(\phi_{SL} - V_{bs}) - 1\}^{\frac{3}{2}} - \{\beta(\phi_{S0} - V_{bs}) - 1\}]^{\frac{3}{2}}$$
$$+ \sqrt{\frac{2\varepsilon_s q N_{sub}}{\beta}}\,[\{\beta(\phi_{SL} - V_{bs}) - 1\}^{\frac{1}{2}} - \{\beta(\phi_{S0} - V_{bs}) - 1\}]^{\frac{1}{2}}$$

電流式は表面ポテンシャルの関数

J. R. Brews, Solid-State Electron. , vol. 21, p.345, 1978;
G. Baccarani, IEEE J. Solid- State Electron Devices, vol. 2, p.62, 1978;
F. van der Wiele, Solid-State Electron., vol.22, p.991, 1979.

ドレイン電流式の比較

Drift-Diffusion Approximation: HiSIM

$$I_{ds} / \left(\frac{1}{\beta} \mu \frac{W}{L} \right) = C_{ox}(1+\beta V'_G)(\phi_{SL} - \phi_{S0}) - \frac{\beta}{2} C_{ox}(\phi_{SL}^2 - \phi_{S0}^2)$$

$$- \frac{2}{3} \sqrt{\frac{2\varepsilon_s q N_{sub}}{\beta}} \left[(\beta \phi_{SL} - 1)^{\frac{3}{2}} - (\beta \phi_{S0} - 1)^{\frac{3}{2}} \right]$$

$$+ \sqrt{\frac{2\varepsilon_s q N_{sub}}{\beta}} \left[(\beta \phi_{SL} - 1)^{\frac{1}{2}} - (\beta \phi_{S0} - 1)^{\frac{1}{2}} \right]$$

$$V'_G = V_{gs} - V_{fb} + (\Delta V_{th})$$

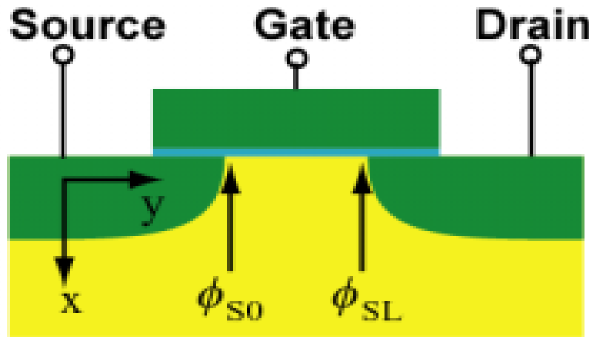
Drift Approximation: $\phi_{S0} = 2\Phi_B = \frac{2}{\beta} \, ln \left(\frac{N_{sub}}{n_i} \right)$; $\phi_{SL} = \phi_{S0} + V_{ds}$

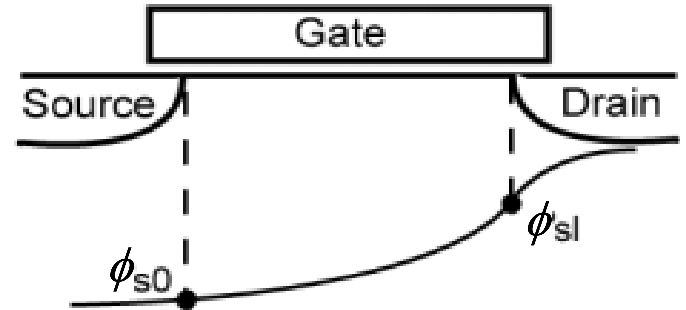
$$I_{ds} / \left(\mu \frac{W}{L} C_{ox} \right) = \left(V_G - 2\Phi_B - \frac{\sqrt{2\varepsilon_s q N_{sub}}}{C_{ox}} 2\Phi_B^{\frac{1}{2}} \right) V_{ds} - \left(\frac{1}{2} + \frac{\sqrt{2\varepsilon_s q N_{sub}}}{4C_{ox}} 2\Phi_B^{-\frac{1}{2}} \right) V_{ds}^2$$

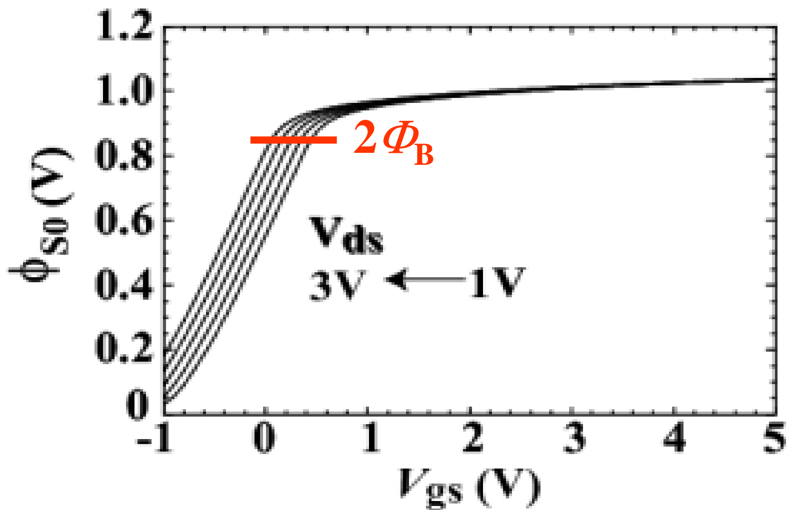
$$\simeq (V_G - V_{th}) V_{ds} - \frac{1}{2} V_{ds}^2$$

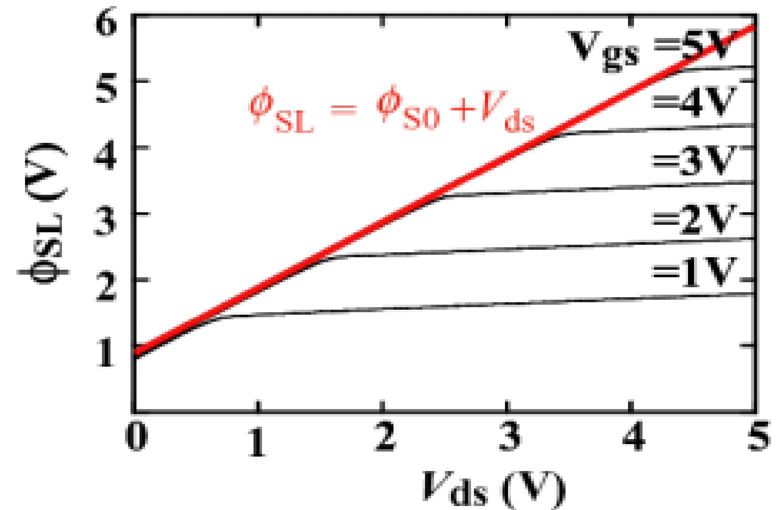
$$V_G = V_{gs} - V_{fb}$$

ドリフト近似の表面ポテンシャル



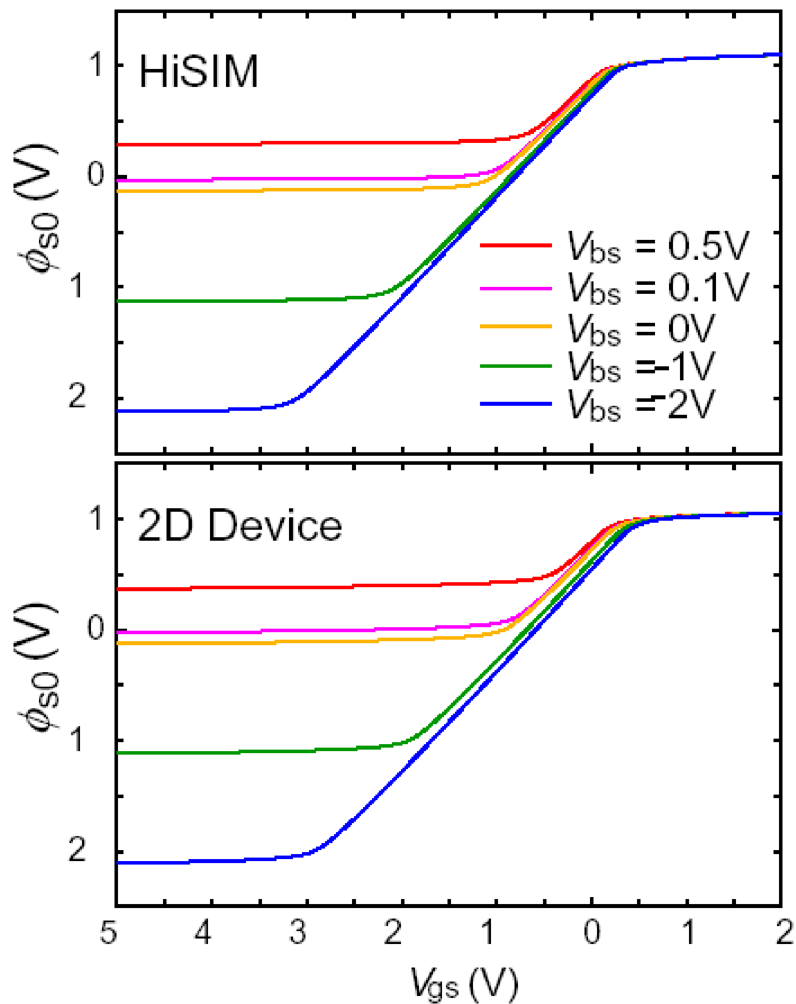


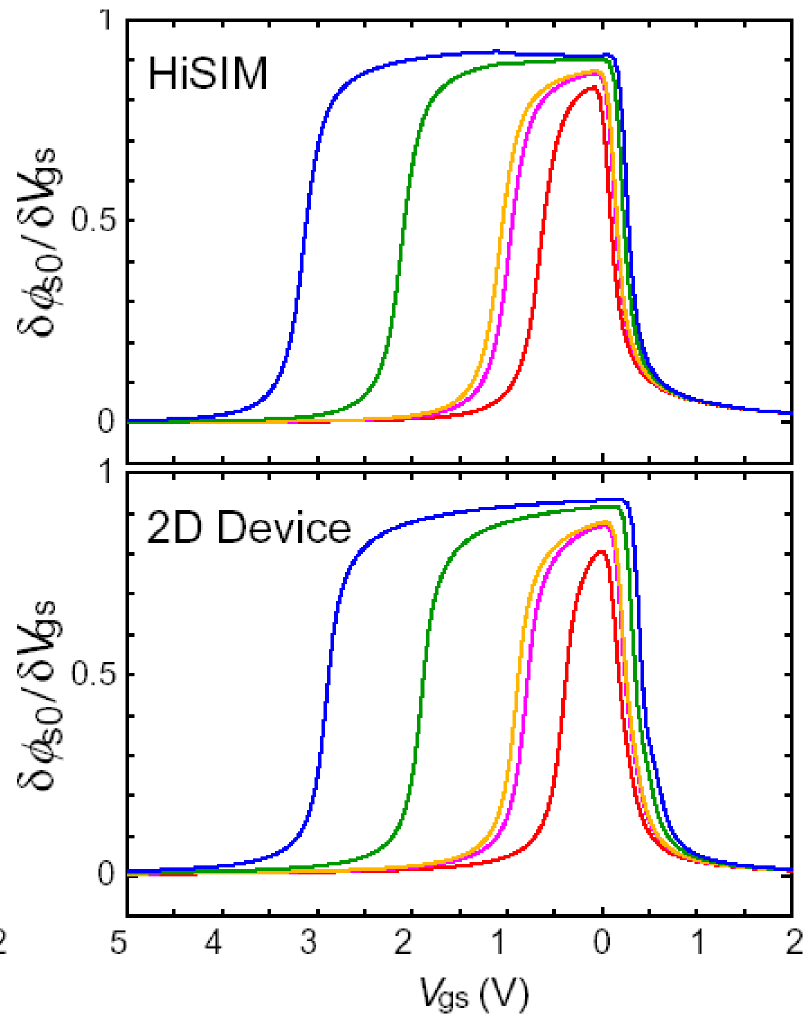




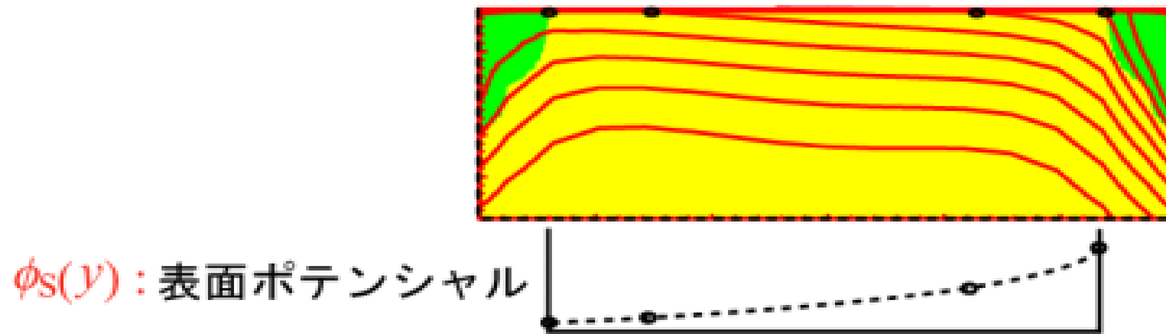
ϕ_{SL}: Gradual-Channel近似の終わり

表面ポテンシャルとその微分





チャネル方向のポテンシャル分布



Charge-Sheet近似

Poisson's Equation:

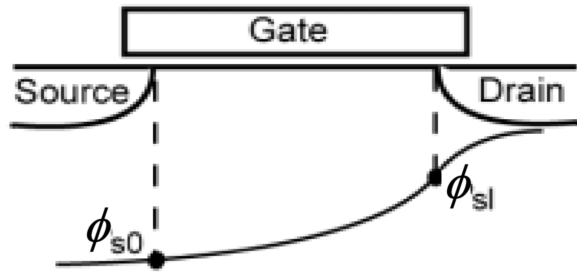
$$Q_{\mathrm{b}}(y) = -qN_{\mathrm{sub}} \times W_{\mathrm{d}} = -\sqrt{\frac{2\varepsilon_{\mathrm{s}}\, qN_{\mathrm{sub}}}{\beta}} \left\{\beta\left(\phi_{\mathrm{s}}(y) - V_{\mathrm{bs}}\right) - 1\right\}^{-\frac{1}{2}}$$

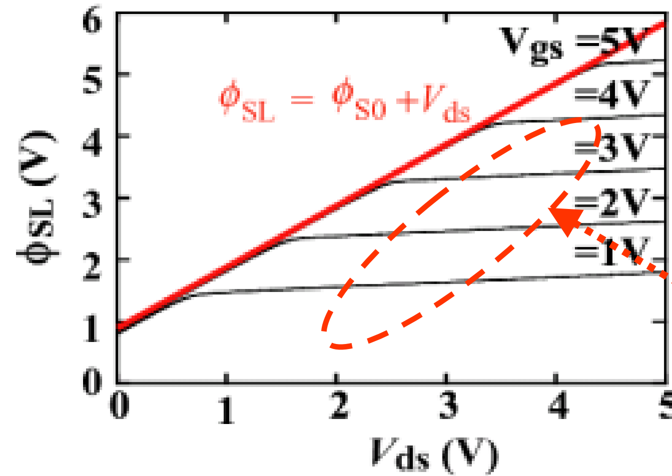
Gauss's Law:

$$Q_{\mathrm{g}}(y) = C_{\mathrm{ox}}\left(V_{\mathrm{G}}' - \phi_{\mathrm{s}}(y)\right)$$

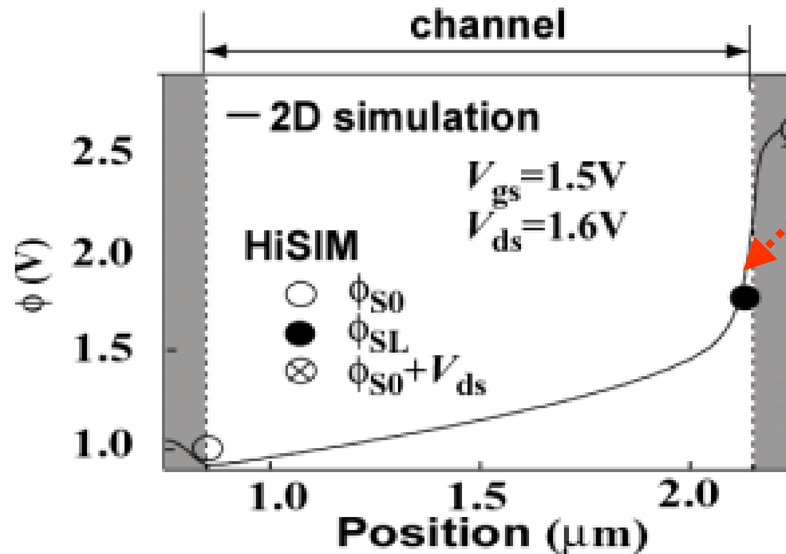
$$Q_{\mathrm{g}}(y) = -\left(Q_{\mathrm{i}}(y) + Q_{\mathrm{b}}(y)\right)$$

Gradual-Channel近似





飽和領域



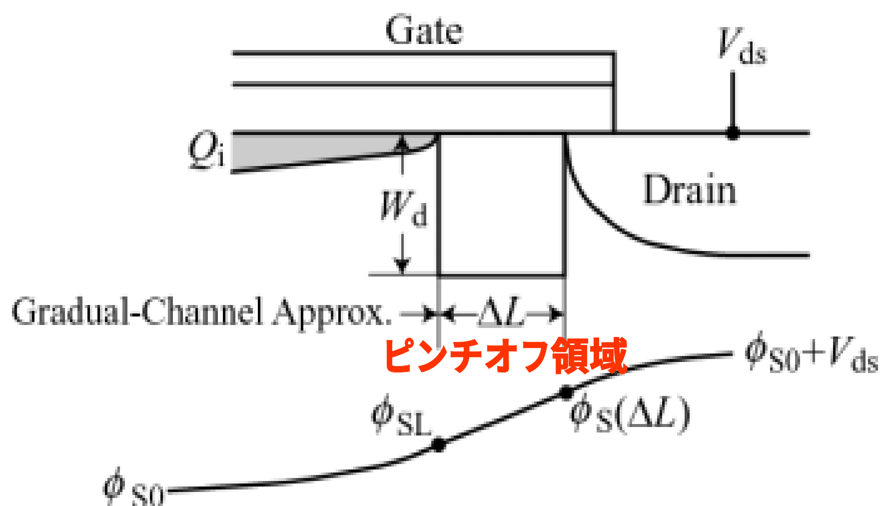
ピンチオフ領域

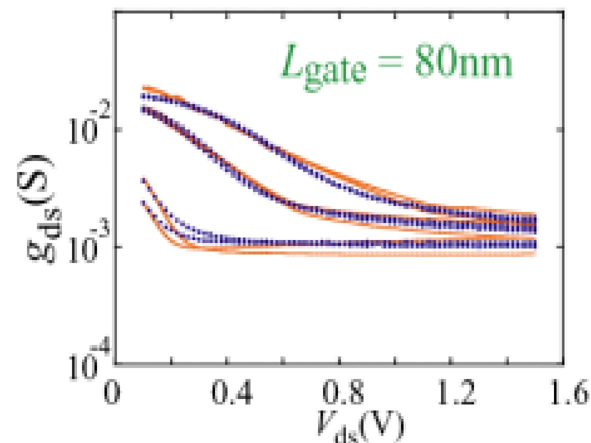
ϕ_{SL} : End of Gradual-Channel Approx.

チャネル内位置？

Channel-Length Modulation

Gradual-Channel 近似の拡張





ポテンシャル値と位置を同時に決定できない。

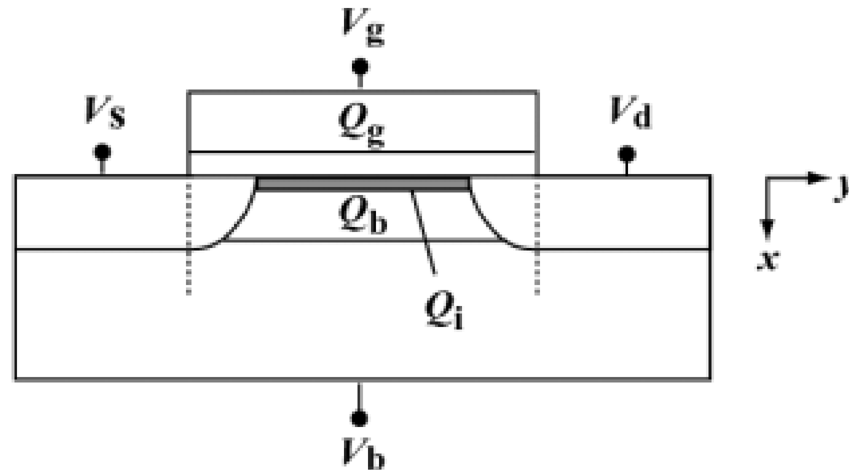
ポテンシャルを定義： $\phi_S(\Delta L) = CLM1\ (\phi_{S0} + V_{ds}) + (1 - CLM1)\ \phi_{SL}$

（*CLM1*はジャンクション形状依存）

$$\Delta L = \varepsilon_S \frac{E(\Delta L) - E(0)}{qN_{sub} + Q_i/W_d}$$

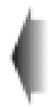
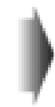
D. Navarro et al., IEICE Trans. Electronics, to be published, 2005.

ノードに誘起される電荷

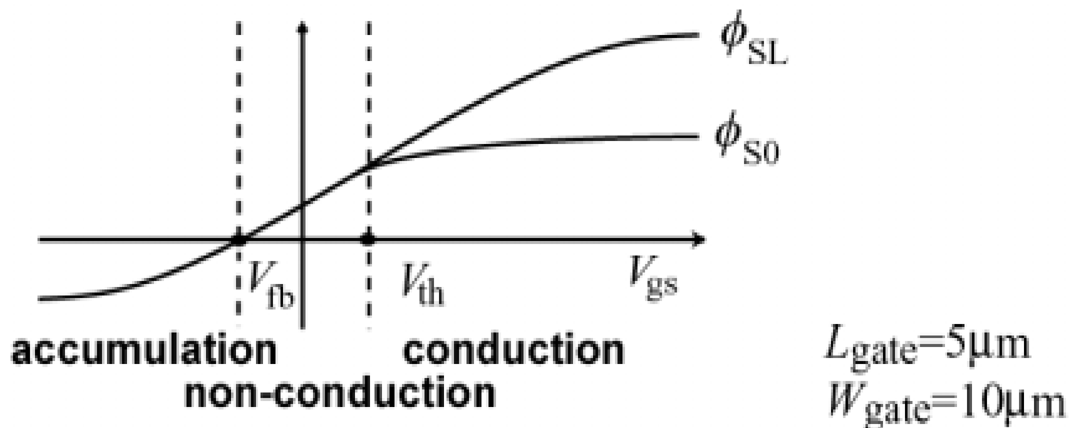


$$Q_I = W\int_0^L Q_i(y)dy; \qquad Q_B = W\int_0^L Q_b(y)dy$$

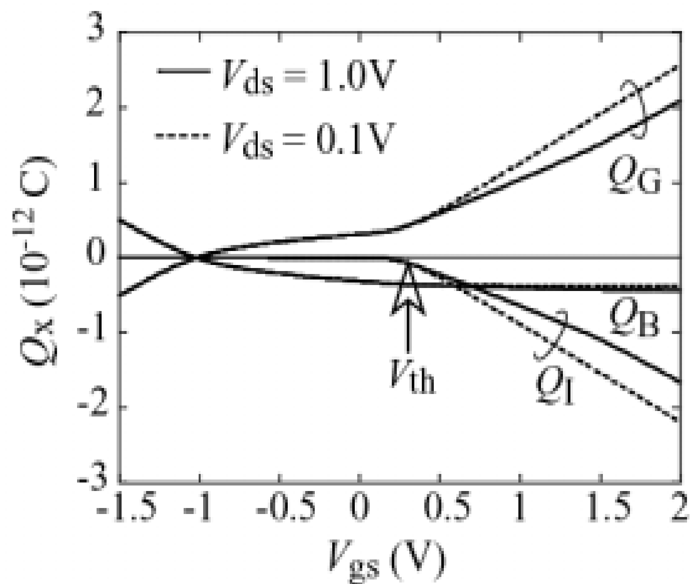
$$Q_S = W\int_0^L \left(1-\frac{y}{L}\right)Q_i(y)dy; \qquad Q_D = W\int_0^L \frac{y}{L}Q_i(y)dy$$

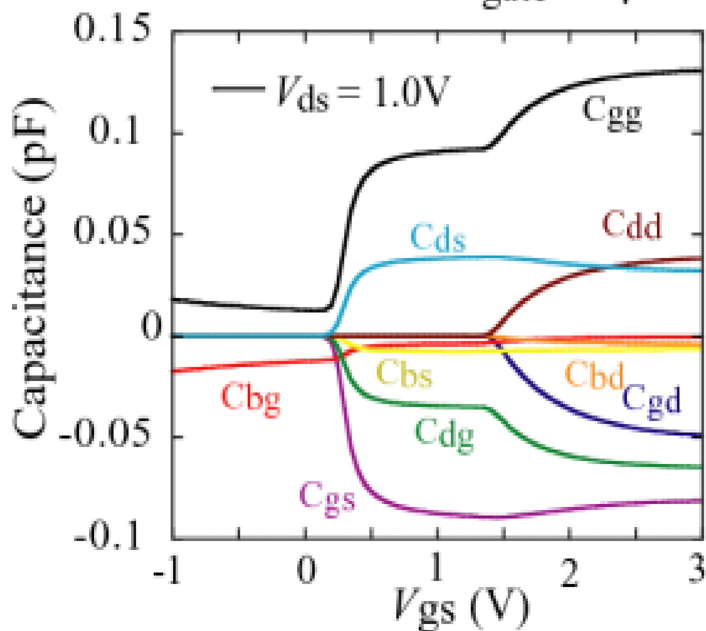
$I_{ds} = q\frac{W}{L}\int vQ_i dy$		Q		$C_{jk}=\frac{dQ_j}{dV_k}$
current		charge		capacitance

電荷とキャパシタンス



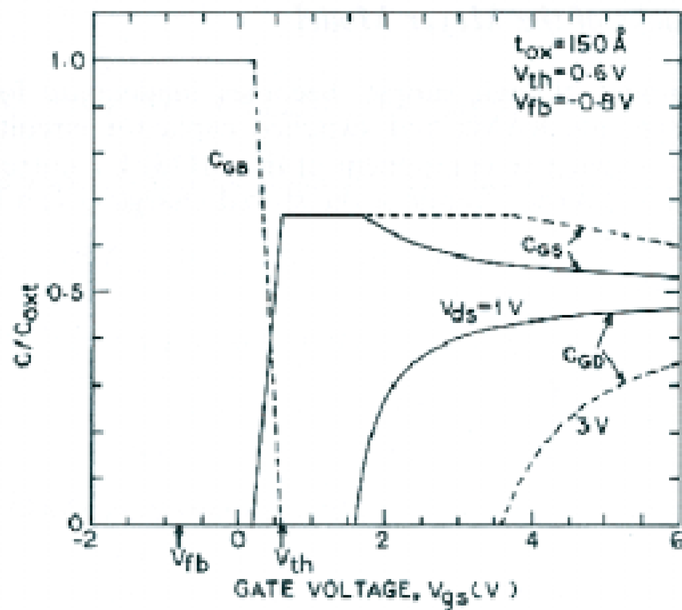
L_{gate}=5µm
W_{gate}=10µm



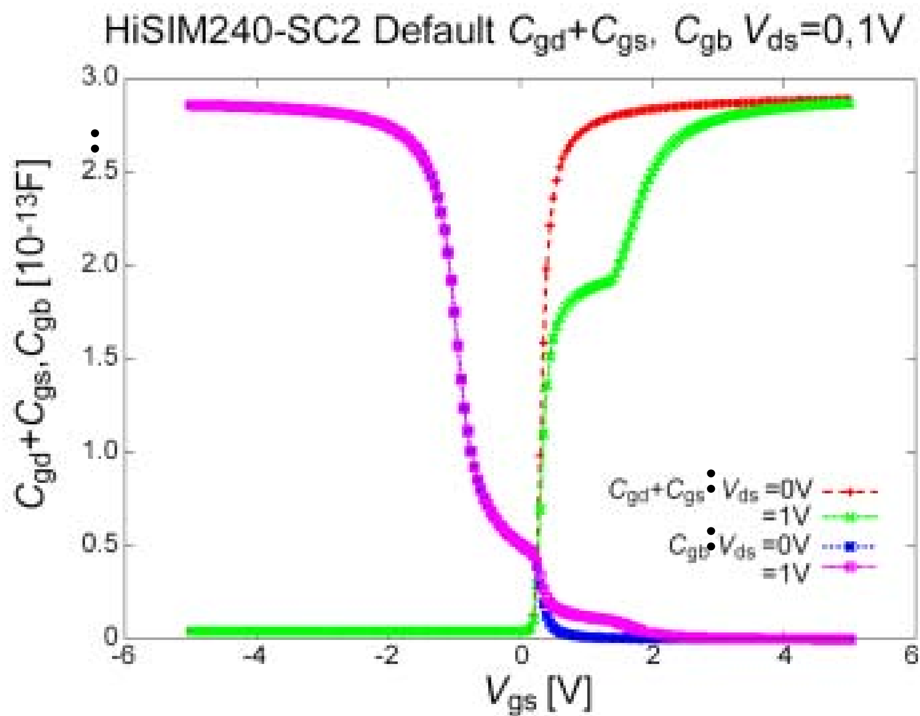


HiSIM User's Manual, http://www.starc.or.jp/kaihatu/pdgr/hisim/index.

Driftモデル

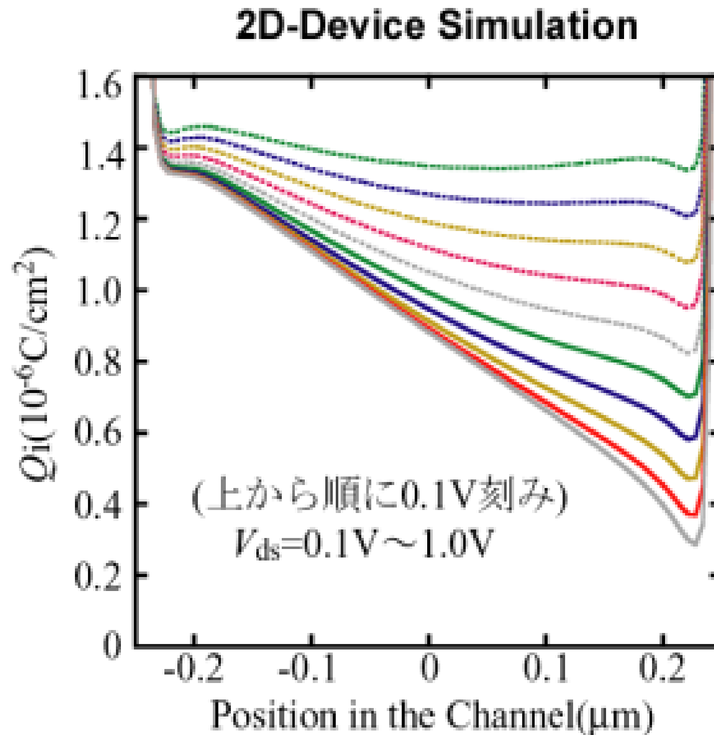


表面ポテンシャルモデル



電荷の分配

$I_x(t) = I_x(V(t)) + dQ_x/dt$



$$Q_I = Q_S + Q_D$$

$$Q_S = W\int_0^L \left(1 - \frac{y}{L}\right) Q_i(y)\,dy$$

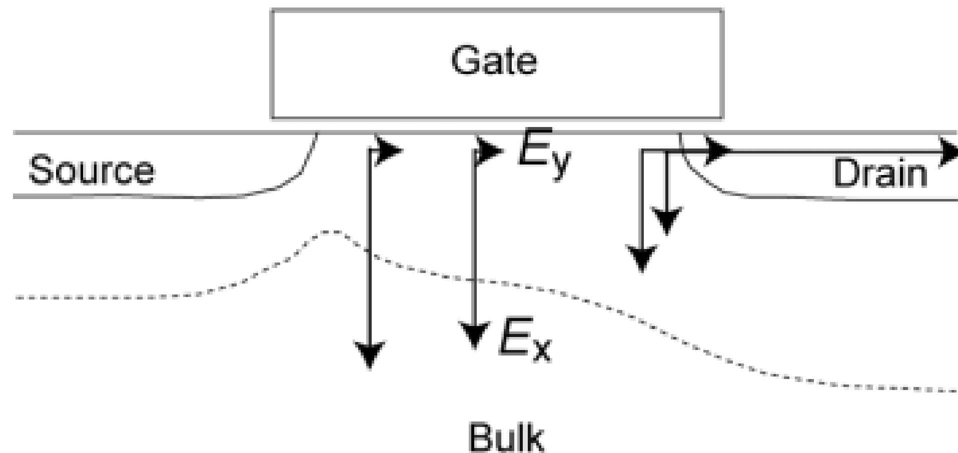
$$Q_D = W\int_0^L \frac{y}{L} Q_i(y)\,dy$$

S. - Y. Oh, D. E. Ward, and R.W. Dutton,
IEEE J. Solid-State Circuits, vol. SC-15, p.636, 1980.

移動度モデル

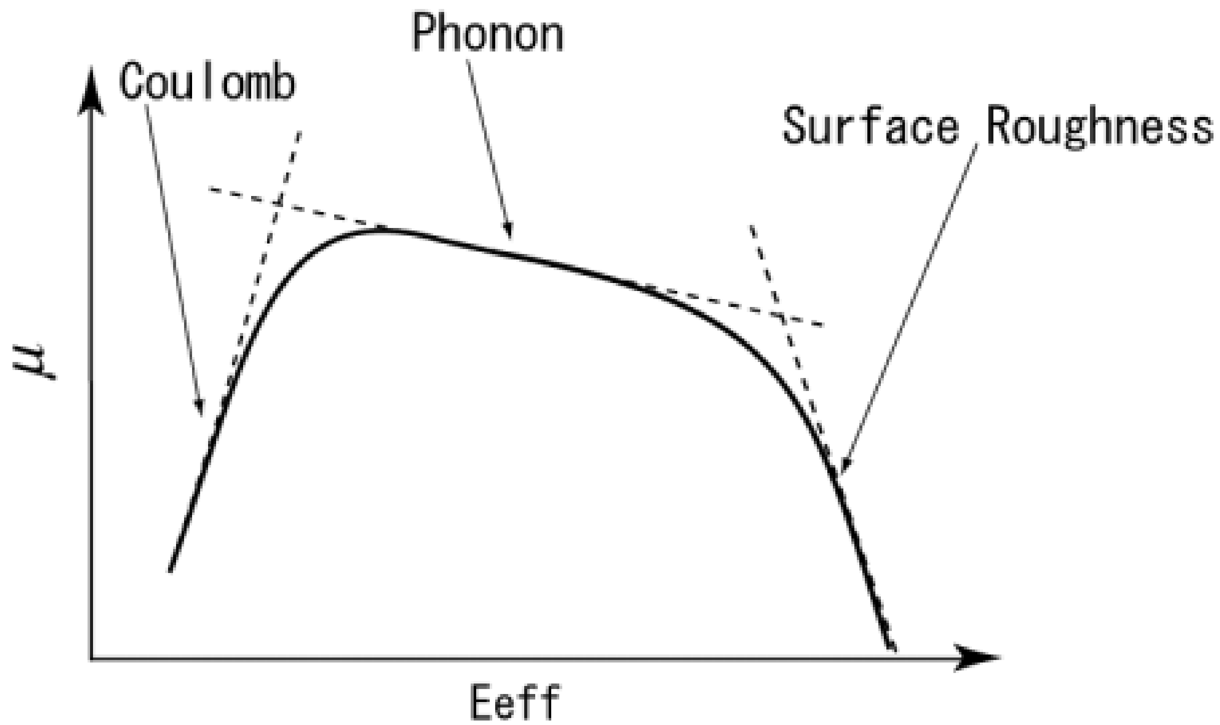
$$I_{ds} = \mu \frac{W}{L} C_{ox} \left[\left(V_{gs} - V_{th} \right) V_{ds} - \frac{1}{2} V_{ds}^{2} \right]$$

μ : mobility (cm^2/Vs)



- **low-field mobility**
- **high-field mobility**

低電界移動度を支配するキャリア散乱



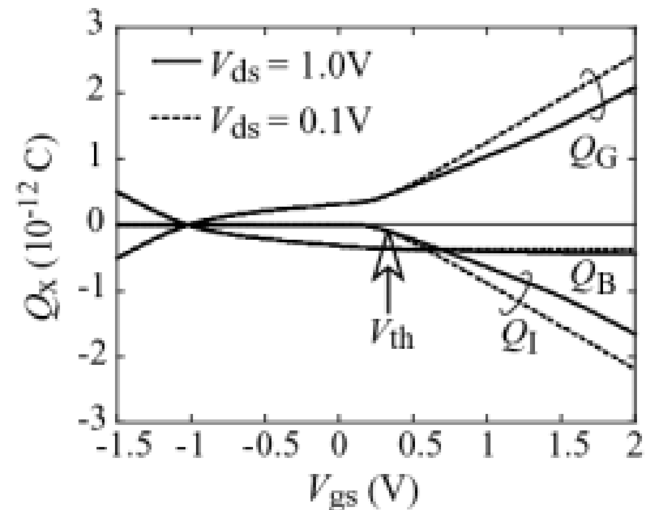
$$\frac{1}{\mu_0} = \frac{1}{\mu_{CB}} + \frac{1}{\mu_{PH}} + \frac{1}{\mu_{SR}}$$ **: Matthiessen Rule**

- μ_{CB} (Coulomb) $= MUECB0 + MUECB1 \dfrac{Q_i}{q \times 10^{11}}$
- μ_{PH} (phonon) $= \dfrac{MUEPH0}{(T/300\,K)^{MUETMP} \times E_{eff}^{MUEPH1}}$
- μ_{SR} (surface roughness) $= \dfrac{MUESR0}{E_{eff}^{MUESR1}}$

$$E_{eff} = \frac{1}{\varepsilon_S}(NDEP \times Q_b + NINV \times Q_i)$$

$$NINV = NINV - NINVD \times V_{ds}$$

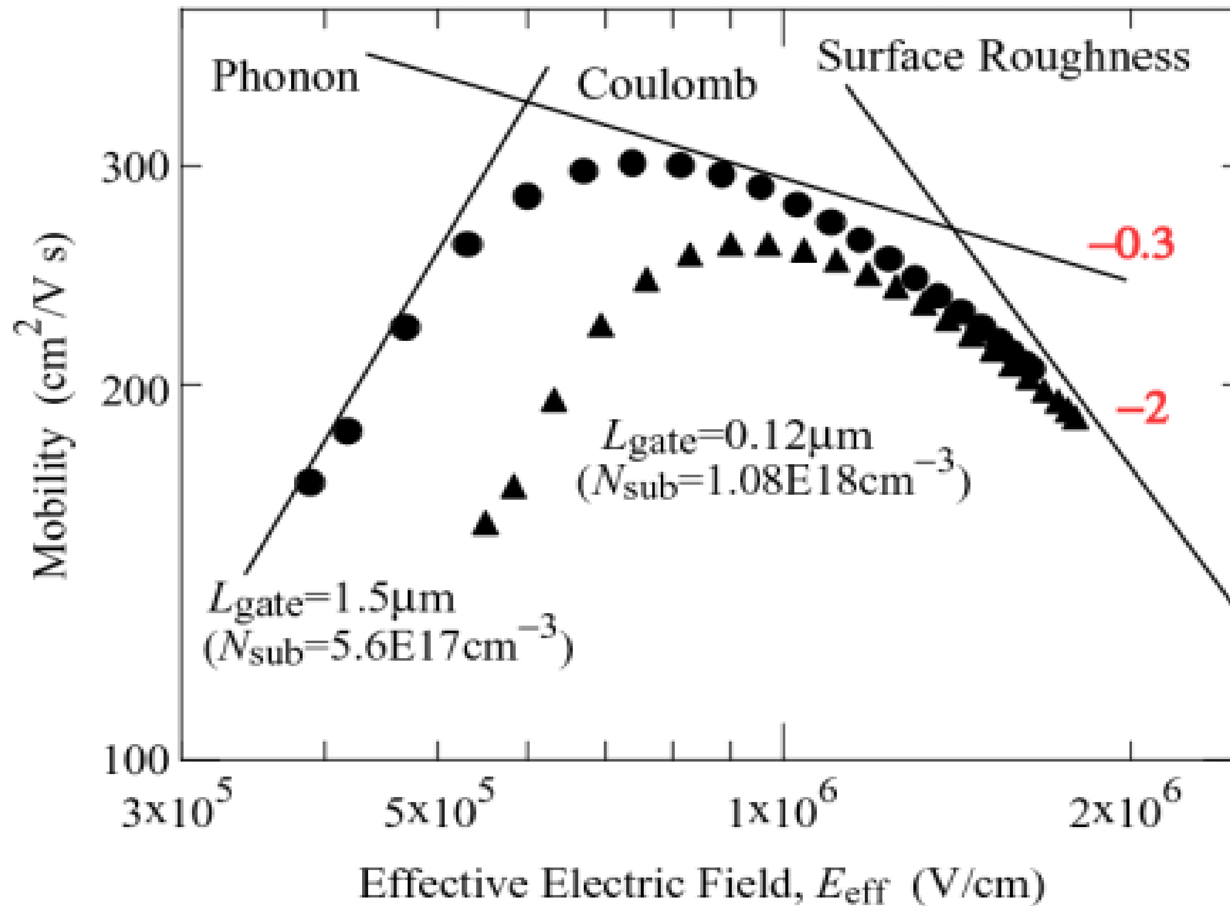
Universality:
$MUEPH1 = 0.3$
$MUESR1 = 2.0$
+
$NDEP = 1.0$
$NINV = 0.5$



E_{eff} はほぼ V_{gs} の線形関数

A.G Sabnis and J. T. Clemens, Tech Dig. IED M, P.18, 1979.

実測値の考察



S. Matsumoto et al., J. Appl. Phys., vol. 92, p. 5228, 2002.

高電界移動度

$$\mu = \frac{\mu_0}{\left(1+\left(\frac{\mu_0 E_y}{VMAX}\right)^{BB}\right)^{\frac{1}{BB}}}$$

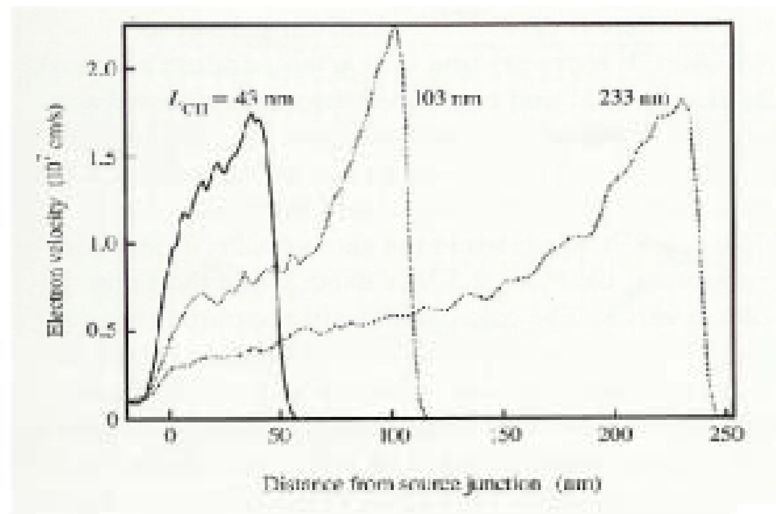
BB = 2.0 (electron)
1.0 (hole)

$$VMAX = \frac{VMAX}{(1.8 + 0.4 \times (T/300\text{K}) + 0.1 \times (T/300\text{K})^2)}$$

:Caughtey-Thomas

$$VMAX = VMAX\ (1.0 + VOVER / \ L_{\text{gate}}{}^{VOVERP})$$

Velocity Overshoot



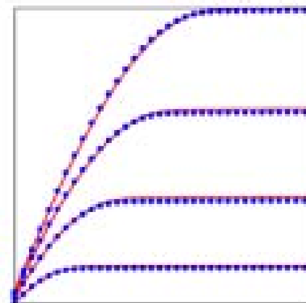
S. E. Laux et al., IBM J. R&D, vol.34, p. 466, 1990.

実測電流値と計算値の比較

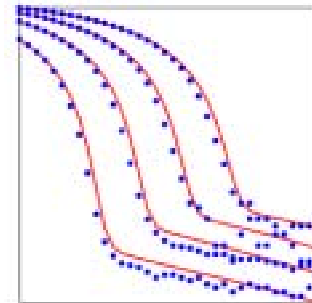
NMOS
Large

HiSIM2
Measurement

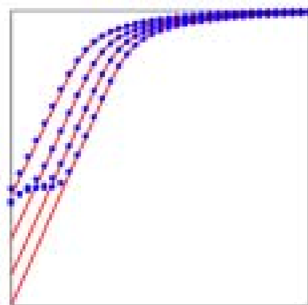
IdVd



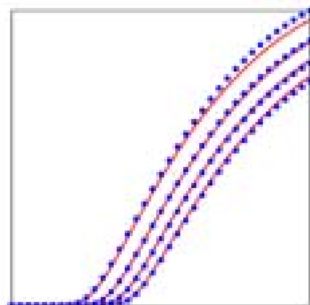
gds



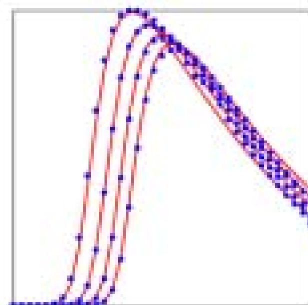
IdVg_low



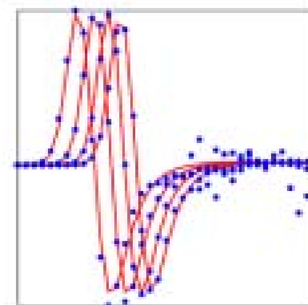
IdVg_low



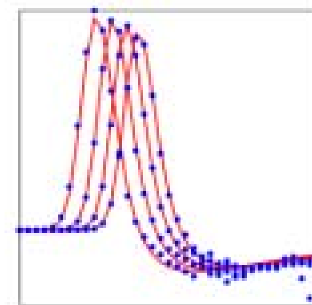
gm_low



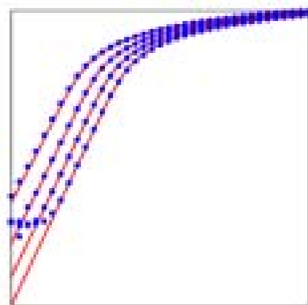
gm2_low



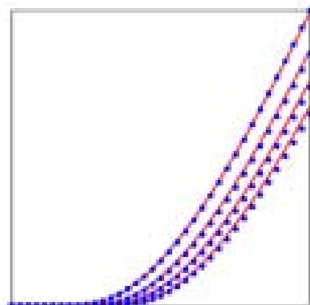
gm3_low



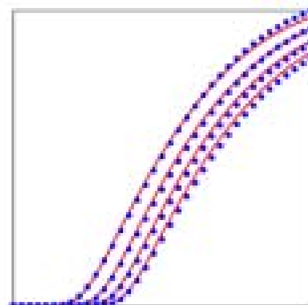
IdVg_high



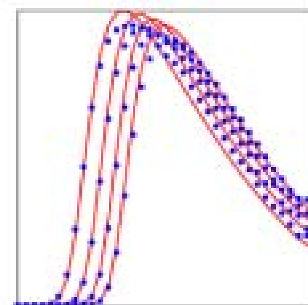
IdVg_high



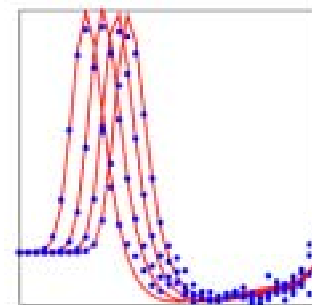
gm_high



gm2_high

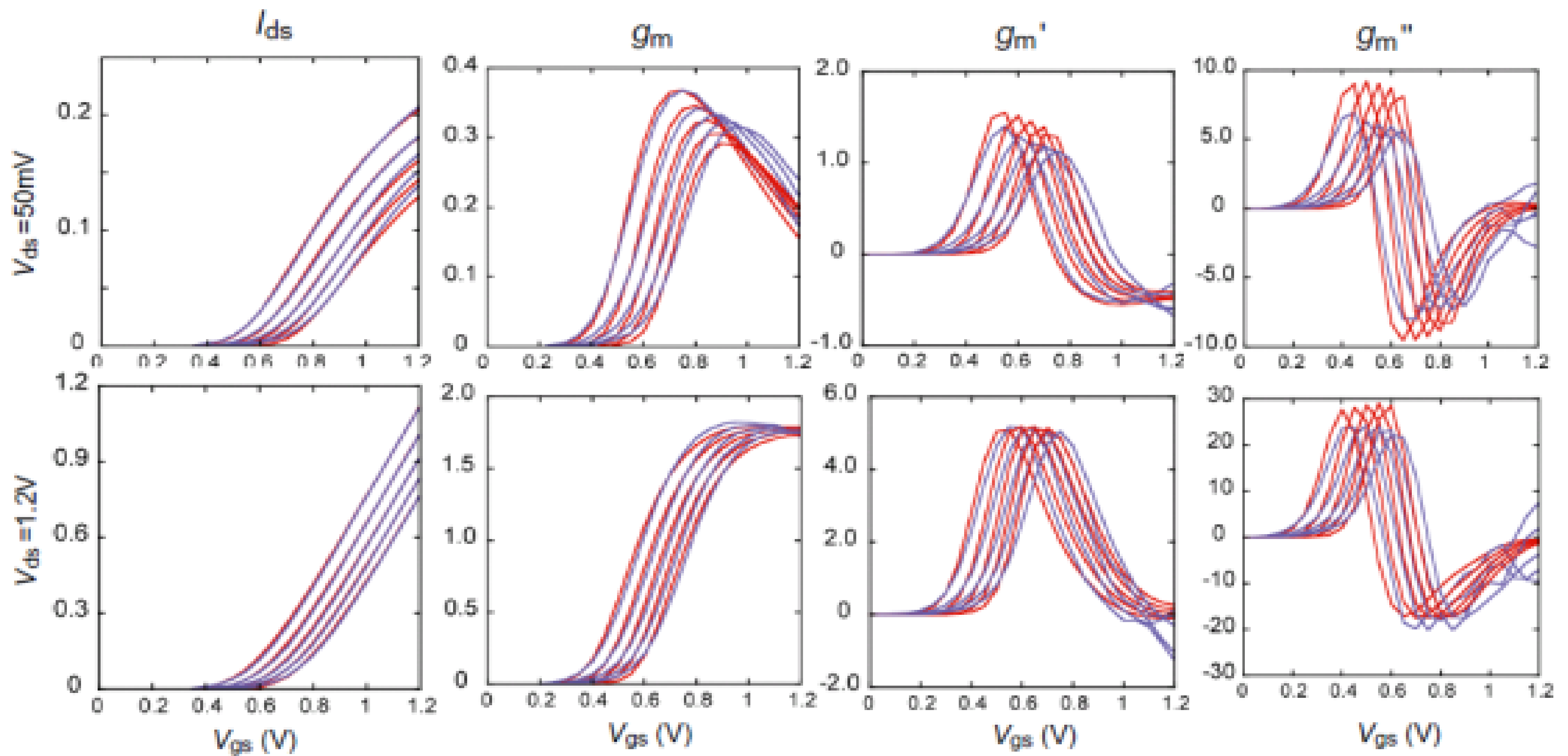


gm3_high

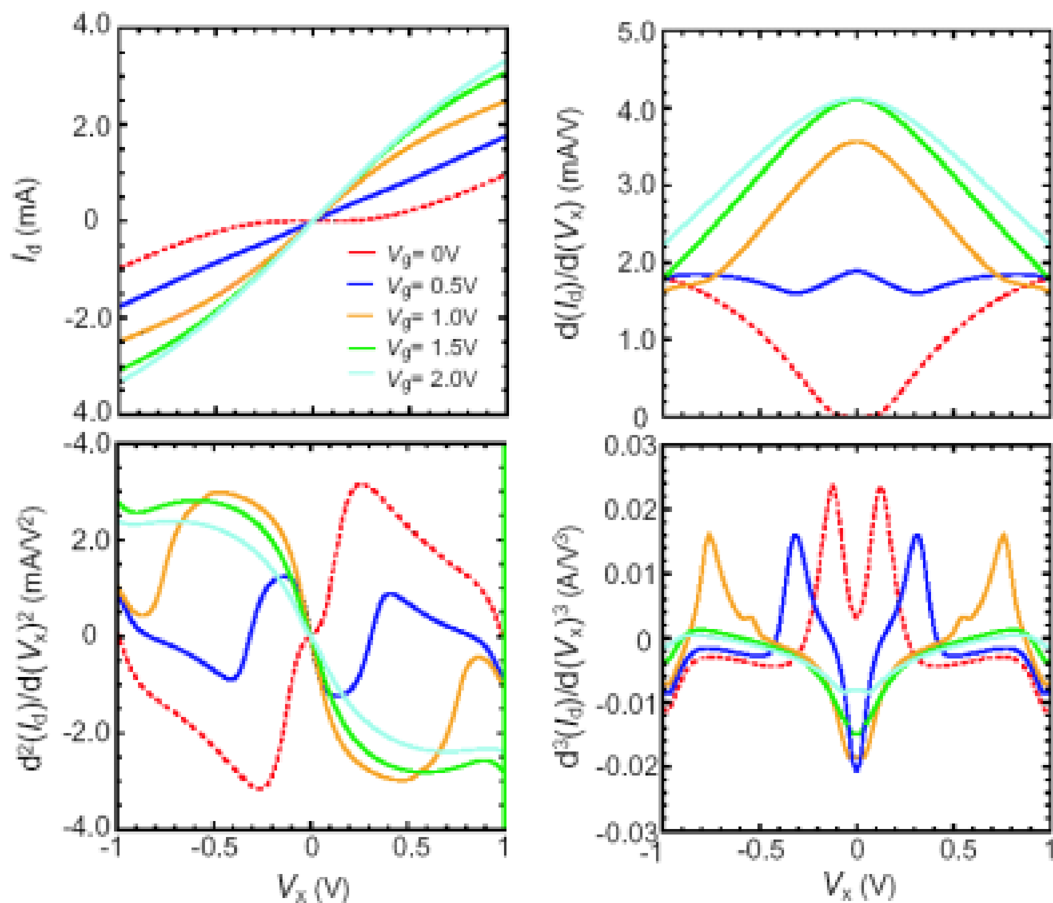


微分特性

L_{gate}=40nm



対称性



ドリフト近似では実現できない

$$I_{ds} = \mu \frac{W}{L} C_{ox} \left[(V_{gs} - V_{th}) V_{ds} - \frac{1}{2} V_{ds}^2 \right]$$

基本方程式

-Poisson:

$$\nabla^2\phi = -\frac{q}{\varepsilon_{Si}}(N_D - N_A + p - n)$$

$$n = n_i \exp\frac{q(\phi - \phi_n)}{kT}$$

$$p = n_i \exp\frac{q(\phi_p - \phi)}{kT}$$

-Current Density:

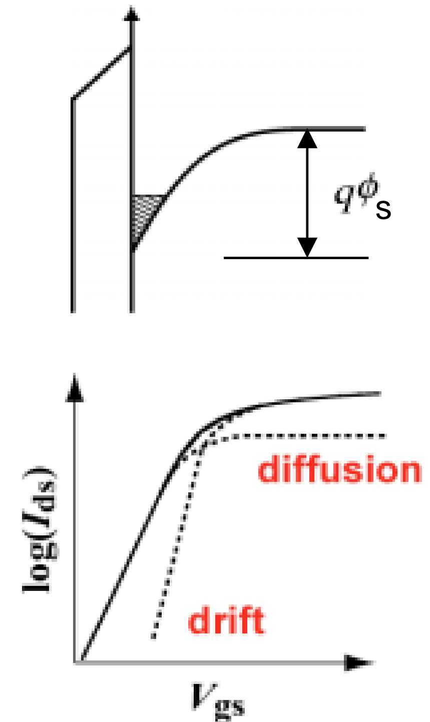
$$j_n = q\mu_n n\frac{\phi}{y} + qD_n\nabla n$$

$$j_p = q\mu_p p\frac{\phi}{y} - qD_p\nabla p$$

-Continuity:

$$I(t) = I_0(t) + \frac{dQ}{dt}$$

(solved by SPICE)



すべてのデバイス特性はポテンシャルの関数

Donald O. Pederson

You don’t get any credit for doing 95 percent of the job.

SPICE: Simulation Program with Integrated Circuit Emphasis

SPICEシミュレーションの基礎

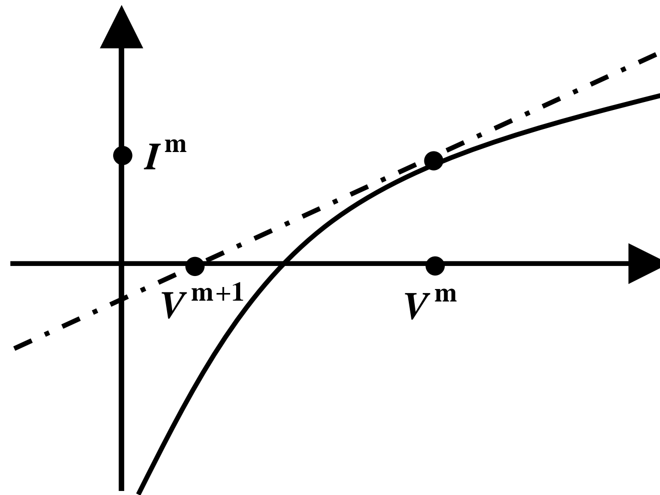
Circuit Simulator ：

Finding a set of $V_s' = (V_1, V_2, V_3, \ldots V_n)$

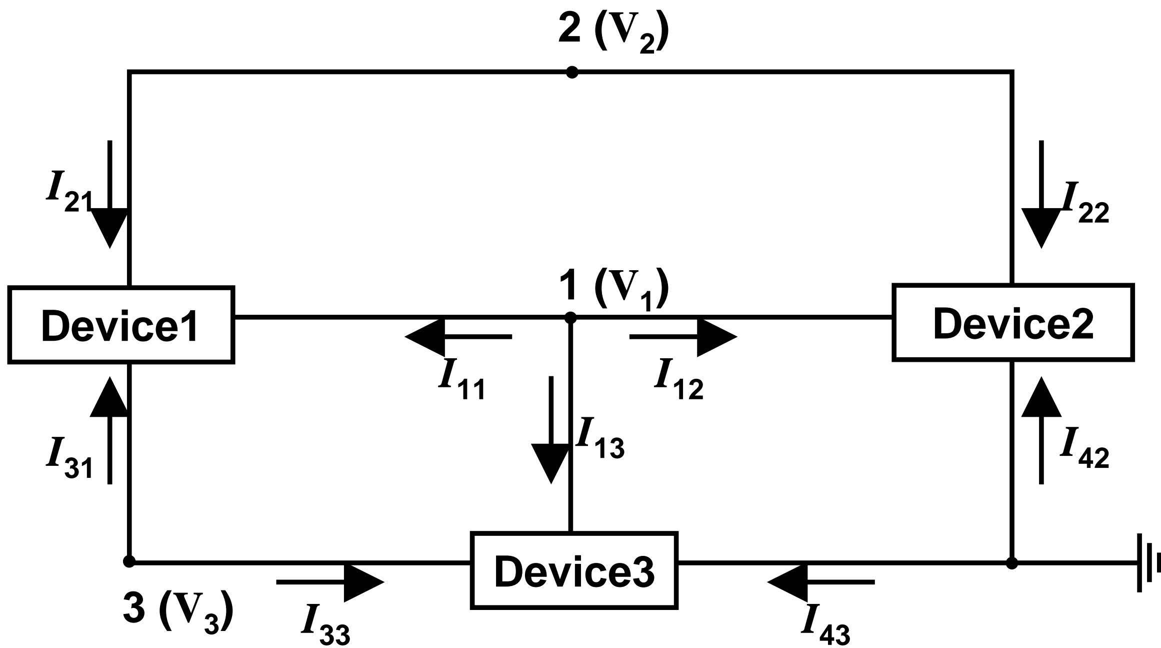
for $I_s'(I_1, I_2, I_3, \ldots I_n)$ by fulfilling the kirchhof law.

$$V^{m+1} = V^m - \frac{I^m}{g^m}$$

$$\Rightarrow g^m V^{m+1} = -I^m + g^m V^m$$



Newton's Method



$$I_1 = I_{11} + I_{12} + I_{13}$$
$$I_2 = I_{21} + I_{22}$$
$$I_3 = I_{31} + I_{13}$$

$$\begin{bmatrix} +\frac{\partial I_{11}^{m}}{\partial V_1} + \frac{\partial I_{11}^{m}}{\partial V_2} + \frac{\partial I_{11}^{m}}{\partial V_3} \\ +\frac{\partial I_{21}^{m}}{\partial V_1} + \frac{\partial I_{21}^{m}}{\partial V_2} + \frac{\partial I_{21}^{m}}{\partial V_3} \\ +\frac{\partial I_{31}^{m}}{\partial V_1} + \frac{\partial I_{31}^{m}}{\partial V_2} + \frac{\partial I_{31}^{m}}{\partial V_3} \end{bmatrix} \begin{bmatrix} V_1^{m+1} \\ V_2^{m+1} \\ V_3^{m+1} \end{bmatrix} = - \begin{bmatrix} +I_{11}^{m} \\ +I_{21}^{m} \\ +I_{31}^{m} \end{bmatrix} + \begin{bmatrix} +\frac{\partial I_{11}^{m}}{\partial V_1} + \frac{\partial I_{11}^{m}}{\partial V_2} + \frac{\partial I_{11}^{m}}{\partial V_3} \\ +\frac{\partial I_{21}^{m}}{\partial V_1} + \frac{\partial I_{21}^{m}}{\partial V_2} + \frac{\partial I_{21}^{m}}{\partial V_3} \\ +\frac{\partial I_{31}^{m}}{\partial V_1} + \frac{\partial I_{31}^{m}}{\partial V_2} + \frac{\partial I_{31}^{m}}{\partial V_3} \end{bmatrix} \begin{bmatrix} V_1^{m} \\ V_2^{m} \\ V_3^{m} \end{bmatrix}$$