Partition Number, Redundant Number and Golden Ratio in Reliability

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Abstract There exist many characteristic functions which are peculiar to reliability theory such as reliability function, failure rate, availability, and so on. However, there exist no special number such as prime number, Fibonacci number, and perfect number in mathematics. This paper attempts to define two numbers which appear often in reliability models.

We have met with some reliability models in our studies whose characteristic values were improved by partitioning their function suitably. Then, we have much experienced that the summation of integers from 1 to N, i.e., $N(N+1)/2 = 1, 3, 6, 10, 15, 21, \ldots$, plays an important role in obtaining optimal policies. We call N, which is a minimum number such that $\sum_{k=1}^{N} k \geq a$ for a parameter a, a *partition number*. As examples of a partition number, we give (1) replacement with a finite interval, (2) network with two terminals, and (3) a standby redundant system.

On the other hand, it has been well-known that system reliability increases usually by redundancy. In particular, the MTTF for a parallel redundant system with exponential failure times is given by the summation of a reciprocal integer, i.e., $\sum_{k=1}^{N}(1/k)$. From this view point, we call N, which is a minimum number such that $\sum_{k=1}^{N}(1/k) \geq a$, a *redundant number*. As examples of a redundant number, we give (4) a parallel system and (5) 1-unit system.

Finally, we give two examples of the golden ration appeared in reliability theory; (6) a series-parallel system and (7) entropy model.